Mth 113 Monday 4-15-13 Class Notes

Inverses of Functions:

Function: a relation (set of ordered pairs) where each x is paired with exactly one y.

Inverse of a function: swap the (x, y) pairs.

Two functions $f(x)$, $g(x)$ are inverses if $f(g(x)) = x$ and $g(f(x)) = x$

(composition of the two functions)

Also written $(f \circ g)(x)$

A function that has an inverse that is also a function is called a one-to-one function ⇒ passes vertical and horizontal line test.

* See page 139 — are these one-to-one?

Inverse functions will also be symmetrical about the line $y = x$

$f^{-1}(x)$ means the inverse function of $f(x)$. x is the independent variable.

Is $y = \sin(x)$ a one-to-one function? If not, how do we "make" $y = \sin^{-1}(x)$ a function.

↓

Also called $y = \arcsin(x)$

↑

read: the angle whose sine is x
or the "arc" sine of x

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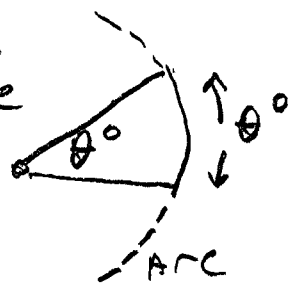
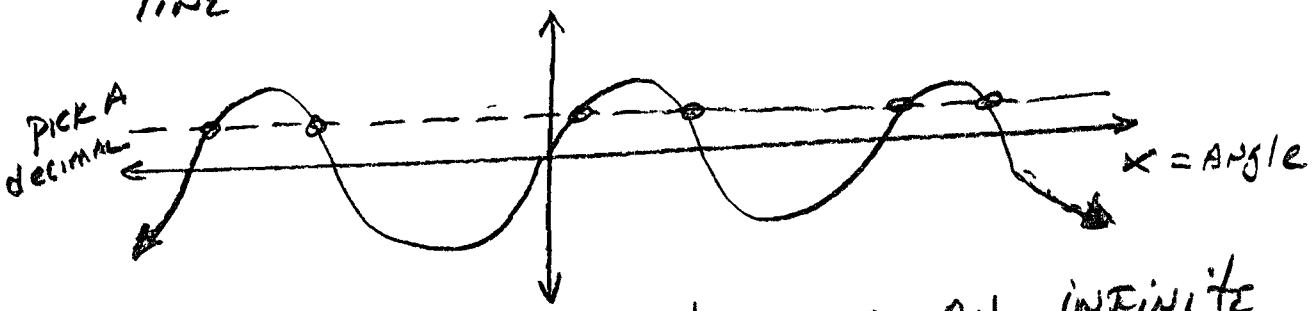
from arc of unit circle



The SINE function is NOT ONE-TO-ONE since it fails the horizontal line test!



PICK A decimal, there ARE AN INFINITE NUMBER OF ANGLES that ARE PAIRED with it ⇒ fails the horizontal line test.

How CAN WE MAKE $y = \sin^{-1}(x)$ work

(y: ↑ ANGLE; x: ↑ decimal)

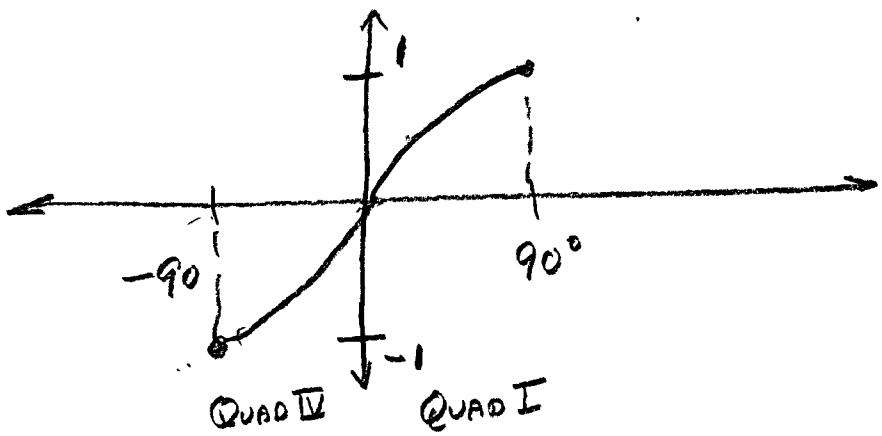
Q: What is the range of $y = \sin(x)$?

A: range is $-1 \leq y \leq 1$

We create A $y = \sin^{-1}(x)$ function by restricting the angles of $y = \sin(x)$ (domain) to keep it ONE-to-ONE but still get All decimals (range).

Lets look at restricting the domain of $y = \sin(x)$ to $-90^\circ \le x \le 90^\circ$ $\left(-\frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}\right)$

and see if ① it is one-to-one
② it include the entire range $(-1 \le y \le 1)$



Pg 142 Inverse Sine Function

$y = \sin^{-1}(x) \Rightarrow$ means

① $\sin y = x$

② $-90^\circ \le y \le 90^\circ$ $\left(-\frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}\right)$

③ $|x| \le 1$ $(-1 \le x \le 1)$

"the $\sin^{-1}(x)$ is the angle in Quadrant I or IV whose sine is x." (decimal)

WolframAlpha computational... knowledge engine

Plot y=sin(x), y=arcsin(x) where (-Pi/2 <= x <= Pi/2) , (-1 <= y <= 1)

Examples Random

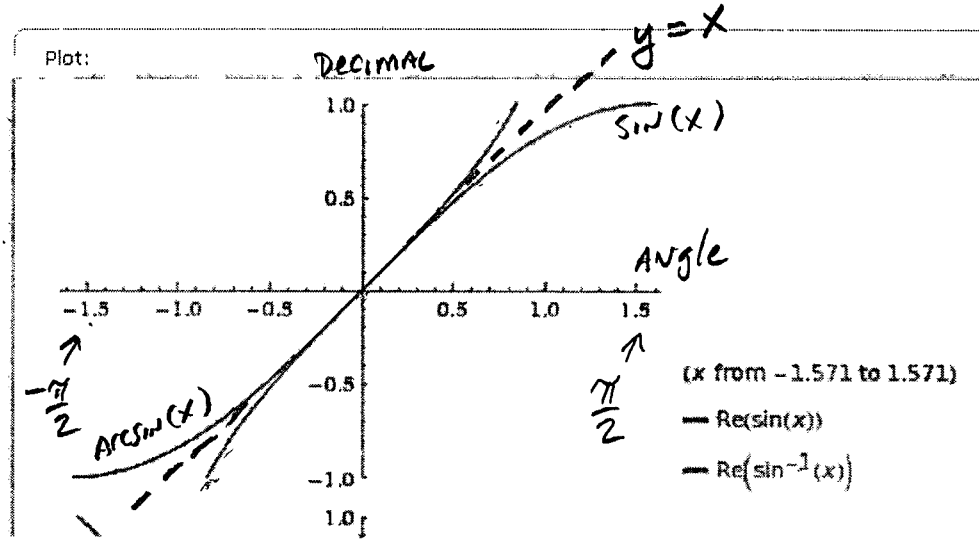
Input interpretation:

plot $y = \sin(x)$ $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$

$y = \sin^{-1}(x)$ $y = -1$ to 1

$\sin^{-1}(x)$ is the inverse sine function »

Plot:



WolframAlpha computational... knowledge engine

Plot y=arcsin(x)

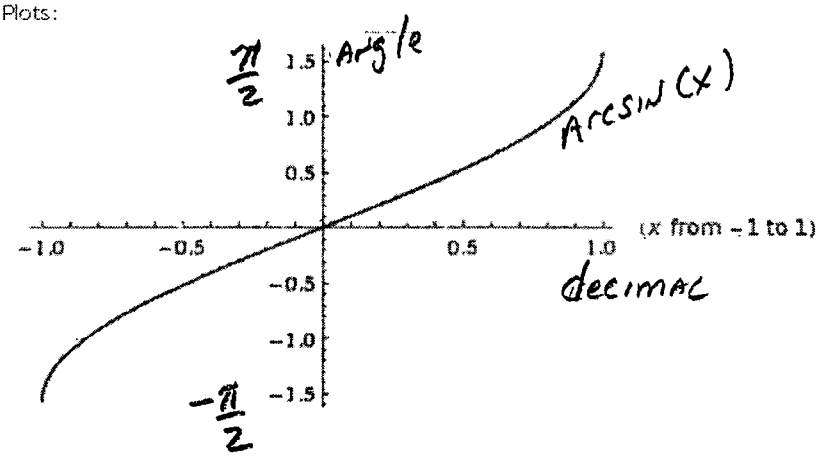
Examples Random

Input interpretation:

plot $y = \sin^{-1}(x)$

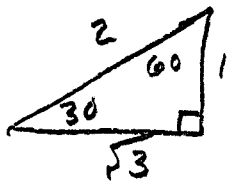
$\sin^{-1}(x)$ is the inverse sine function »

Plots:



Enable interactivity

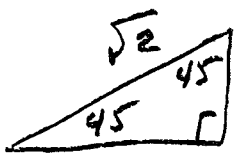
(Ex 1) Pg 143 $\sin^{-1}\left(\frac{1}{2}\right) = ?$ radians, deg.



$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{30^\circ = \frac{\pi}{6} \text{ rad}}$

↑ QUAD I

(Ex 2) Pg 144 $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right) = ?$ rad, deg



S	A
T	C

$\Rightarrow$ QIV

NOTE: $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-45^\circ = -\frac{\pi}{4} \text{ rad}}$

(Ex 1) Pg 144 $\sin^{-1}(.5312) = ?$ rad., deg.

QI

$\Rightarrow \boxed{\theta = 32.0866^\circ = .1783\pi \text{ rad} \text{ or } \approx .5600 \text{ rad}}$

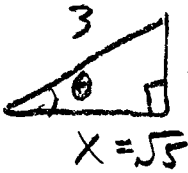
(Ex 2) Pg 144 $\text{Arcsin}(-.9249) = ?$

QIV

$\Rightarrow \boxed{\theta = -67.6533^\circ = -.3759\pi \text{ rad} \text{ or } -1.1808 \text{ rad}}$

(Ex1) pg145 Simplify $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

@ I



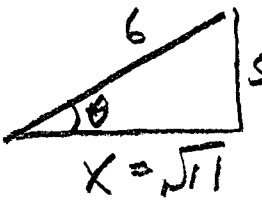
$\sin^{-1}\left(\frac{2}{3}\right) \Rightarrow x = \sqrt{9-4} = \sqrt{5}$

$\therefore \tan(\text{angle } \theta \text{ that has a sin of } \frac{2}{3})$

$= \tan(\theta) = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$

(Ex2) pg145 $\sec\left[\arcsin\left(-\frac{5}{6}\right)\right]$

@ IV



$x = \sqrt{36-25} = \sqrt{11}$

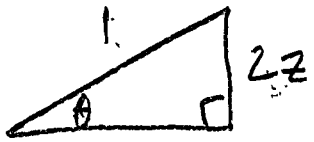
$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{a} = \frac{6}{\sqrt{11}} = \boxed{\frac{+6\sqrt{11}}{11}} = \theta$

@ IV

sec = +

(Ex3) pg145 $\tan(\arcsin(2z))$, $z > 0$

$\Rightarrow$ QUAD I



$x = \sqrt{1^2 - (2z)^2}$

$= \sqrt{1 - 4z^2} \Rightarrow$ DOS

$= \sqrt{(1-2z)(1+2z)}$

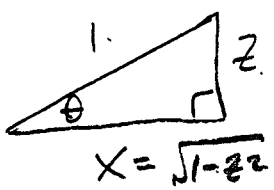
$\therefore \tan\theta = \frac{2z}{\sqrt{1-4z^2}} \cdot \frac{\sqrt{1-4z^2}}{\sqrt{1-4z^2}} = \frac{2z\sqrt{1-4z^2}}{1-4z^2}$

$\tan\theta = \frac{2z\sqrt{1-4z^2}}{(1-2z)(1+2z)}$	$z > 0$

WE WILL RATIONALIZE THE DENOMINATOR ☺

(Ex4) pg146 $\cos(\sin^{-1}(z))$, $z < 0$

@ IV

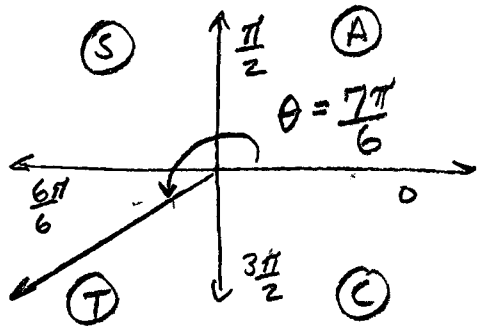


$x = \sqrt{1^2 - z^2} = \sqrt{1-z^2}$ DOS

$\cos\theta = \frac{\sqrt{1-z^2}}{1} = \sqrt{(1-z)(1+z)}$ $\quad z < 0$

cos + in @ IV

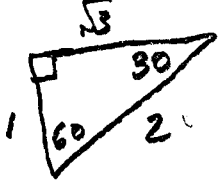
(Ex5) Pg 147 $\sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$



$\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$

$\theta' = \frac{\pi}{6} = 30°$



$\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-30°}$

QIV

WARNING $\sin^{-1}[\sin(x)] = x$

(See pg 146)

iff $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$