Mth 113 Weds. 2-13-13 Class Notes

Verifying Trig. Identities involving Double & Half Angles

(Ex 1) Pg 189

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{2\tan\theta}{\sec^2\theta}$$

$$= \frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = \frac{2\sin\theta\cos^2\theta}{\cos\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \quad \checkmark$$

But $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(EX 2) $\sin 4\theta$

PG 189

USING

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\sin 4\theta$

$= 2\sin 2\theta \cos 2\theta$

$2\left[(2\sin\theta\cos\theta)(2\cos^2\theta - 1)\right]$

USING

$\cos 2\alpha = 2\cos^2\alpha - 1$

$= (4\sin\theta\cos\theta)(2\cos^2\theta - 1)$

$= 8\sin\theta\cos^3\theta - 4\sin\theta\cos\theta$

$= 8\sin\theta\cos^3\theta - 4\sin\theta\cos\theta$

$= 4\sin\theta\cos\theta(2\cos^2\theta - 1)$

$= 4\sin\theta\cos\theta((\cos^2\theta - 1) + (\cos^2\theta - 1))$

$= 4\sin\theta\cos\theta(\sin^2\theta + \sin^2\theta)$

$= 4\sin\theta\cos\theta(2\sin^2\theta)$

$= 8\sin^3\theta\cos\theta$

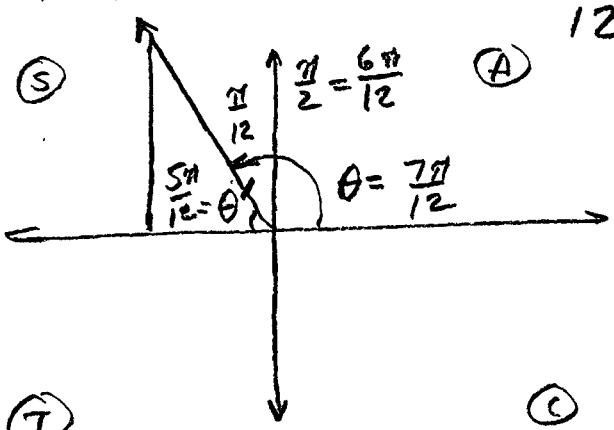
$= 8\sin\theta\cos\theta(1 - \cos^2\theta)$

✓

Mth 113 Weds. 2-13-13 CLASS NOTES

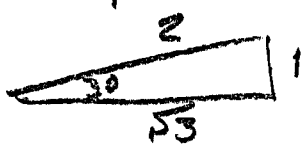
WORKSHEET PRACTICE

(9.) EXACT VALUE: $\tan\frac{7\pi}{12}$



$\theta=\frac{7\pi}{12}\cdot\frac{180^\circ}{\pi\,\text{rad}}=105^\circ$

$\theta'=75^\circ$



$\tan 75^\circ = -\tan(45^\circ+30^\circ)$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$\therefore \tan 75^\circ = -\left[\dfrac{\tan 45+\tan 30}{1-\tan 45\tan 30}\right]$

($\tan 75^\circ$ = $\tan 105^\circ$)

$$=-\left[\frac{1+\frac{1}{\sqrt{3}}}{1-(1)\frac{1}{\sqrt{3}}}=\frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right]$$

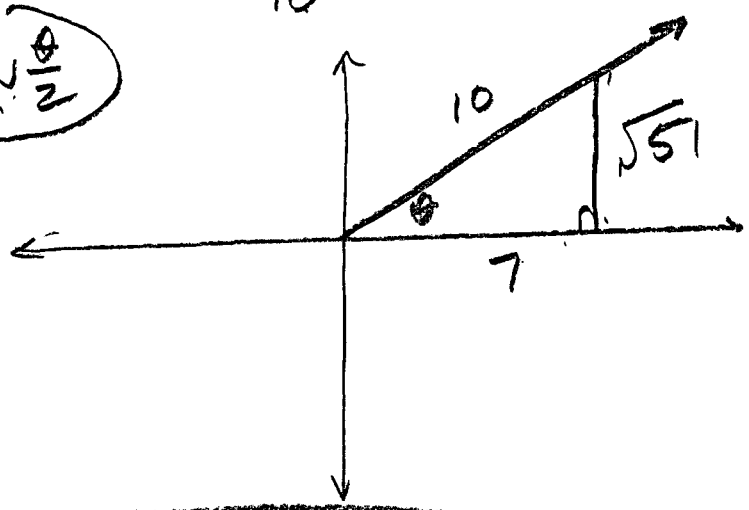
$$=-\left[\frac{\sqrt{3}+1}{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)}\right]=-\left[\frac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}\right]$$

$$=-\left[\frac{3+2\sqrt{3}+1}{3-1}\right]=-\left[\frac{4+2\sqrt{3}}{2}\right]$$

$$\tan\frac{7\pi}{12}=-\tan 75^\circ=-2-\sqrt{3}$$

(33) $\cos\theta = \frac{7}{10}$ $\quad 0° < \theta < 90°$

FIND $\tan\frac{\theta}{2}$



$\frac{\theta}{2}$ in QUAD I
$\therefore +$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$= \sqrt{\frac{1-\frac{7}{10}}{1+\frac{7}{10}}} = \sqrt{\frac{\frac{3}{10}}{\frac{17}{10}}}$$

$$= \sqrt{\frac{3}{17}}$$

$$= \frac{\sqrt{3}}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}}$$

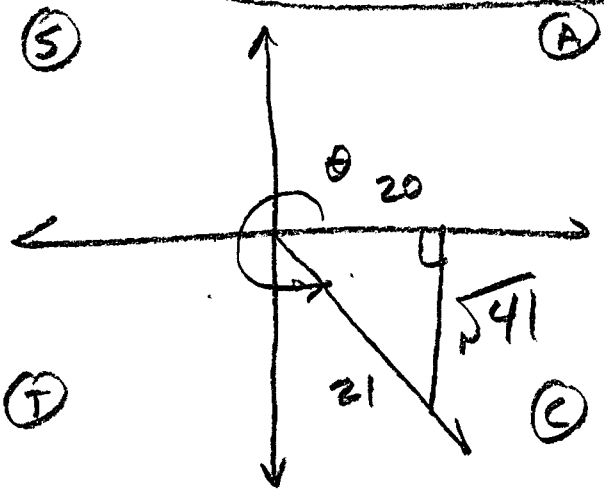
$$\boxed{\tan\left(\frac{\theta}{2}\right) = \frac{\sqrt{51}}{17}}$$

(44) $\cos\theta = \frac{20}{21}$ $\quad 270^\circ < \theta < 360^\circ$

5

$\cos 2\theta = ?$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$



21
21
21
42
441 − 400 = 41

$= \left(\frac{20}{21}\right)^2 - \left(\frac{\sqrt{41}}{21}\right)^2$

$= \frac{400}{441} - \frac{41}{441} = \frac{359}{441}$

$\cos 2\theta = \frac{359}{441}$

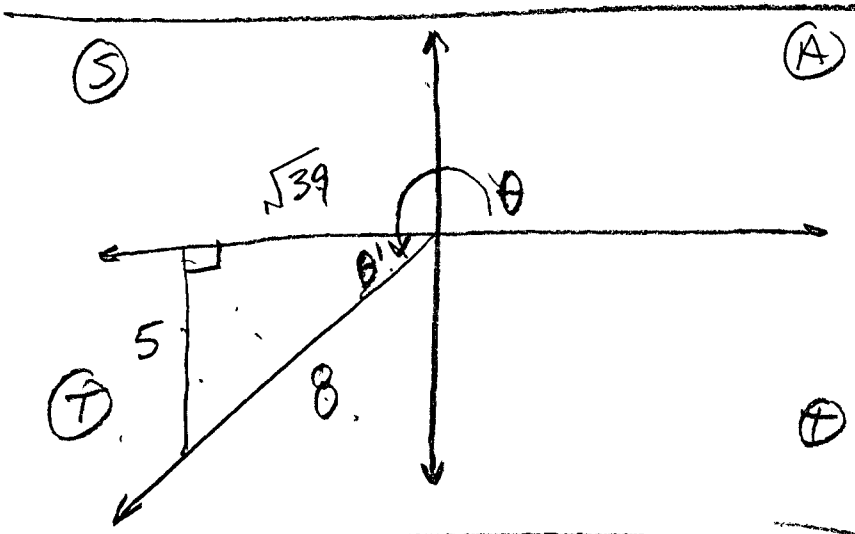
Homework #35 Pg 190

$\csc\theta = -\frac{8}{5} = \frac{u}{10}$ $\pi < \theta < \frac{3\pi}{2}$

QUAD. III

$\frac{1}{\sin}$

$\sin 2\theta,\ \cos 2\theta,\ \tan 2\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= -2\left(\frac{5}{8}\right)\left(-\frac{\sqrt{39}}{8}\right) = +\frac{10\sqrt{39}}{64}$

$= \boxed{\frac{5\sqrt{39}}{32}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{\sqrt{39}}{8}\right)^2 - \left(-\frac{5}{8}\right)^2$

$\frac{39}{64} - \frac{25}{64} = \frac{14}{64} = \boxed{\frac{7}{32}}$

$\tan 2\theta = \frac{\frac{5\sqrt{39}}{32}}{\frac{7}{32}} = \boxed{\frac{5\sqrt{39}}{7}}$