Mth 113 Monday 2-11-13 Class Notes

Ch. 5-2 The Sum and Difference Identities AND Cofunction Identities

Sum and Difference Identities

SUM	• $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (see Appendix C)
DIFF	• $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
SUM	• $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
DIFF	• $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
SUM	• $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
DIFF	• $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Cofunction Identities

Complementary angles add to 90° or $\frac{\pi}{2}$ rad

• $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$	• $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$
• $\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$	• $\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$
• $\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$	• $\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

(Ex)

$\sin 50^\circ = \cos 40^\circ$

$\sec\frac{\pi}{6} = \csc\frac{\pi}{3}$

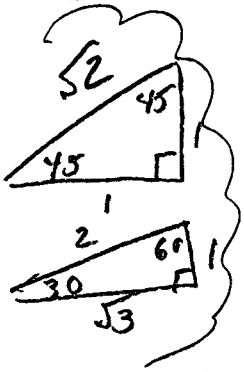
$\uparrow \quad \uparrow$

$30^\circ \quad 60^\circ$

First use of the Sum/Difference Identities ⇒ Finding "EXACT" VALUES of the trig. functions for NON-QUADRANTAL Angles, i.e., ones that are NOT 30°, 45°, 60° or quadrantal.

EX 5-2c pg 180

Find the EXACT VALUE of tan 15°



Since $15° = 45° - 30°$, use the TANGENT DIFFERENCE Identity

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

$$\therefore \tan(45° - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \cdot \frac{1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{1^2 - 2\left(\frac{1}{\sqrt{3}}\right) + \frac{1^2}{3}}{1^2 - \frac{1^2}{3}}$$

$$= \frac{\frac{4}{3} - \frac{2}{\sqrt{3}}}{\frac{2}{3}} = 2 + \frac{3}{\sqrt{3}} = 2 - \frac{3\sqrt{3}}{3}$$

$$= \boxed{2 - \sqrt{3}} \ *$$

* CK w/ TI 30XS

Another type of sum/diff ID problem:

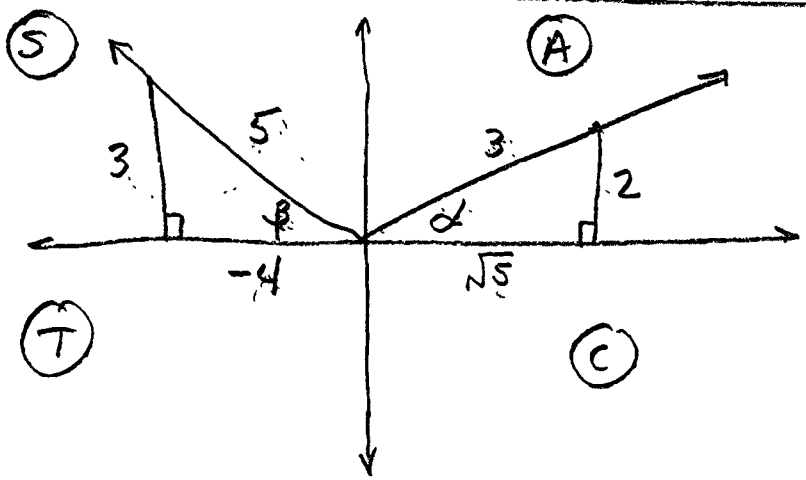
EX 5-2D
Pg 181

$\sin\alpha = \frac{2}{3}$ in Quadrant I

$\cos\beta = -\frac{4}{5}$ in Quadrant II

Find: exact value of $\cos(\alpha - \beta)$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



$$\therefore \cos(\alpha-\beta) = \frac{\sqrt{5}}{3}\cdot -\frac{4}{5} + \frac{2}{3}\cdot\frac{3}{5}$$

$$= -\frac{4\sqrt{5}}{15} + \frac{6}{15}$$

$$= \boxed{\frac{6-4\sqrt{5}}{15}}$$

Cofunction Problems

"the trig. function of θ = "CO" function of its complementary angle

(Ex) $\sin 40^\circ = \cos 50^\circ$

$\tan 20 = \cot 70$

$\sec \frac{\pi}{6} = \csc \frac{2\pi}{6} = \csc \frac{\pi}{3}$

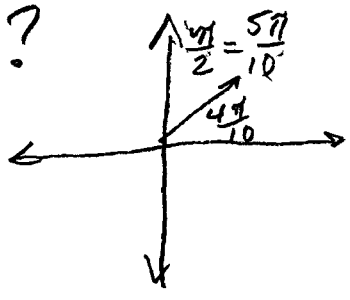
* $\cot 130 = \tan(-40)$ *$130 - 40 = 90$

Rewrite in terms of cofunction - Pg 181

① $\sin 34 = ?$ $\cos 56^\circ$

② $\csc \frac{2\pi}{5} = ?$

$\frac{2\pi}{5} = \frac{4\pi}{10}$



$\sec \frac{\pi}{10}$

Simplify ⇒ using Cofunctions!
pg 181

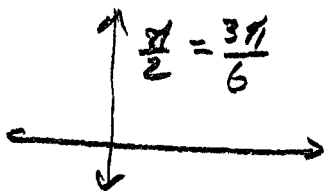
(3) $\frac{\sin 10^\circ}{\cos 80^\circ} = \frac{\sin 10^\circ}{\sin 10^\circ} = \boxed{1}$

COFUNCTION

COFUNCTION

or $\frac{\sin 10^\circ}{\cos 80^\circ} = \frac{\cos 80^\circ}{\cos 80^\circ} = \boxed{1}$

(4) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3}$



$\sin^2 \frac{\pi}{6} + \sin^2 \frac{2\pi}{6}$

$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} = \boxed{1}$

YOUR OWN HANDWRITING

3"

NAME

THE SIX SUM/DIFF IDENTITIES

BACK IS BLANK

5"

* MAY USE FOR QUIZ. If you forget it, or think you might forget it, memorize the 6 identities.