Mth112 Tuesday 10-30-12 Class Notes

Ch 10-2 Rational Zeros of a Polynomial Function

A way to find their actual numerical value, not just how many there are.

⇒ Rational Zero Theorem

* must clear fractions

Given: $a_N x^N + a_{N-1} x^{N-1} + \dots a_1 x + a_0$ where $a_N \to a_0$ are integers

1. if $\frac{p}{q}$ is a rational zero
2. then p is a factor of a_0
3. and q is a factor of a_N

Note: $\dfrac{\text{some factors of } a_0}{\text{some factors of } a_N}$ will not be zeros

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Ex 10-2-A pg 448

$$P(x) = \frac{1}{2}x^4 - \frac{2}{3}x^3 + 2x^2 - \frac{1}{3}x + 2$$

$$0 = \frac{1}{2}x^4 - \frac{2}{3}x^3 + 2x^2 - \frac{1}{3}x + 2$$

Mult. by 6 to get integer coefficients

$$0 = 3x^4 - 4x^3 + 12x^2 - 2x + 12$$

a_N, factors $\pm 1, \pm 3$ → q

a_0, factors $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ → p

∴ possible $\frac{p}{q}$ zeros

Let $q = \pm 1 \Rightarrow \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

Let $q = \pm 3 \Rightarrow \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}$

How many of these work? Descartes' Rule of Signs

EX 2 (pg 449) List all possible Rational Zeros of $P(x) = x^4 + x^3 + x^2 + 3x - 6$

$a_N = 1$ $\qquad$ $a_0 = -6$

$q \Rightarrow$ FACTORS ± 1 $\qquad$ $p \Rightarrow$ FACTORS $\pm 1, \pm 2, \pm 3, \pm 6$

$\therefore \frac{p}{q} \Rightarrow \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}$

$\Rightarrow \boxed{-1, 1, -2, +2, -3, +3, -6, +6}$

Descartes' ROS $\Rightarrow + + + + -$

CHANGE $\Rightarrow$ 1 positive real zero

$P(-x) \Rightarrow + - + - -$

CHANGE CHANGE CHANGE $\Rightarrow$ 3 or 1 negative real zeros

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Check if $x = -1$ is a zero

$\Rightarrow$ is $[x - (-1)]$ a factor $= x + 1$

Polynomial Long Division

$$
\begin{array}{r l}
 & x^3 + 0 + x + 2 \\
x+1 \,\big) & x^4 + x^3 + x^2 + 3x - 6 \\
 & -(x^4 + x^3) \\
 & \quad 0 + x^2 + 3x \\
 & \quad -(x^2 + 1x) \\
 & \qquad 2x - 6 \\
 & \qquad -(2x + 2) \\
 & \qquad\quad -8
\end{array}
$$

Remainder, NOT A FACTOR

Synthetic Division

ADD

$$
\begin{array}{r|rrrrr}
-1 & 1 & 1 & 1 & 3 & -6 \\
 & & -1 & 0 & -1 & -2 \\
\hline
 & 1 & 0 & 1 & 2 & -8
\end{array}
$$

MULTIPLY

Bring 1st Down

Check $X = +1$ $\Rightarrow$ is $(X-1)$ a factor?

$$
\begin{array}{r|rrrrr}
1 & 1 & 1 & 1 & 3 & -6 \\
 & & 1 & 2 & 3 & 6 \\
\hline
 & 1 & 2 & 3 & 6 & 0
\end{array}
$$

Yes, $X = +1$ is a factor

$1x^3 + 2x^2 + 3x + 6$ is how many time $X-1$ goes into $P(x)$

STARTS ONE Degree Lower THAN $P(x)$

Check

$$
\begin{array}{r|l}
 & x^3 + 2x^2 + 3x + 6 \\
\hline
x-1 & x^4 + x^3 + x^2 + 3x - 6 \\
 & -(\cancel{x^4} - x^3) \\
 & \quad 2x^3 + x^2 \\
 & \quad -(\cancel{2x^3} - 2x^2) \\
 & \qquad 3x^2 + 3x \\
 & \qquad -(\cancel{3x^2} - 3x) \\
 & \qquad\quad 6x - 6 \\
 & \qquad\quad -(\cancel{6x} - 6) \\
 & \qquad\qquad 0 \checkmark
\end{array}
$$

Now that you know $(X-1)$ goes into $P(x)$

$\Rightarrow (X-1)(x^3 + 2x^2 + 3x + 6) = P(x)$

you can check the next factors by dividing into either the 3RD degree factor (one less term) or into $P(x)$

LETS try both ways for the synthetic division practice

$P(x) = (x-1)(x^3 + 2x^2 + 3x + 6)$

$x = +1$ = positive zero, looking for 3 or 1 negative zeros.

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orig. possible $\Rightarrow -1, 1, -2, +2, -3, +3, -6, +6$

-1: DID NOT WORK; 1: WORK

$+2, +3, +6$: No more positive zeros possible By DROS

LEFT TO try, $-2, -3, -6$ — ONE OR ALL OF [illegible]K THESE MUST WORK

Pick one! Try -3

$$\begin{array}{r|rrrr} -3 & 1 & 2 & 3 & 6 \\ & & -3 & 3 & -18 \\ \hline & 1 & -1 & 6 & \text{NO} \end{array} \qquad \begin{array}{r|rrrrr} -3 & 1 & 1 & 1 & 3 & -6 \\ & & -3 & 6 & -21 & 54 \\ \hline & 1 & -2 & 7 & -18 & \text{NO} \end{array}$$

$\therefore$ -2 or -6 must work AND THE remaining zeros are non-real and complex

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Try $-2 \Rightarrow [x-(-2)]$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 3 & 6 \\ & & -2 & 0 & -6 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

Yes $(x+2)$ is A Factor

0x

$(1x^2 + 3)$ is remainder

So: $(x-1)(x+2)(x^2+3) = P(x)$

$$(x-1)(x+2)(x^2+3) = 0$$

For zeros

$x=1$
$x=-2$

ZPP

$$x^2+3=0$$

$$x^2=-3$$

$$x=\pm i\sqrt{3}$$

$\therefore P(x) = x^4+x^3+x^2+3x-6$

HAS zeros of

$$\boxed{\{x \mid x = -2, 1, -i\sqrt{3}, i\sqrt{3}\}}$$

WolframAlpha computational knowledge engine

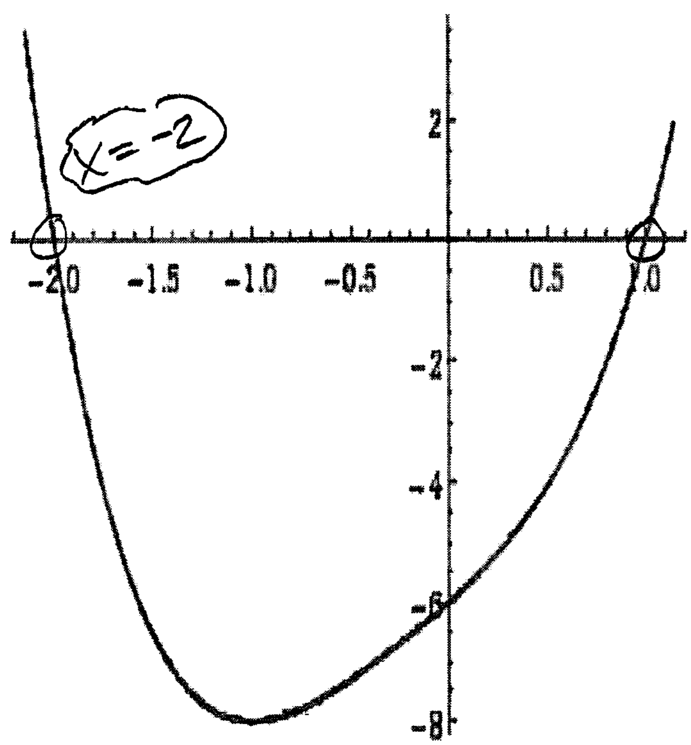
Plot P(x) = x^4 +x^3 +x^2 +3x -6

Examples Random

Input interpretation:

plot	$P(x) = x^4 + x^3 + x^2 + 3x - 6$

Plots:



(x from -2 to 1)

Enable interactivity