Mth 112 Monday 10-29-12 Class Notes

Ch. 10-1 Des Cartes' Rule of Signs

Polynomial Function Pg 438
$$f(x) = a_N x^N + a_{N-1} x^{N-1} + \dots a_{N-2} x^{N-2} + a_1 x + a_0$$

where: $a_N \neq 0$

$a_N, a_{N-1}, \dots a_0$ Are Complex or real

NOTE CZT

Polynomial Equation $f(x) = 0$

Solutions to the PE are the "zeros of the polynomial"

Fundamental Theorem of Algebra

A polynomial function has at least one and at most the same number of complex roots as its degree.

Note
Double root ⇒ multiplicity of 2 $(x-1)(x-1)=0$
triple root ⇒ "A zero of multiplicity 3" $(x-1)(x-1)(x-1)=0$

CZT Complex Zero Theorem

Given a polynomial function $P(x)$ having only <u>real</u> coefficients, if $a+bi$ is a zero, then so is $a-bi$.

COMPLEX CONJUGATES

EX 10-1A Given: zeros of $+1$, $2-i$

PG 440 Find: $P(x)$

$$P(x) = [x-(+1)][x-(2-i)][x-(2+i)]$$

Rearrange and be alert to DOS pattern

Conjugate

$$= (x-1)[(x-2)+i][(x-2)-i]$$

$$(a+b)(a-b) = a^2-b^2$$

$$= (x-1)((x-2)^2-i^2)$$

NOTE $-i^2 = +1$

$$= (x-1)(x^2-4x+5)$$

$$= x^3 - 4x^2 + 5x$$
$$\quad - x^2 + 4x - 5$$

$$\boxed{P(x) = x^3 - 5x^2 + 9x - 5}$$

Any multiple of $P(x)$ has same zeros

WolframAlpha computational knowledge engine

Plot P(x) = x^3-5x^2+9x-5

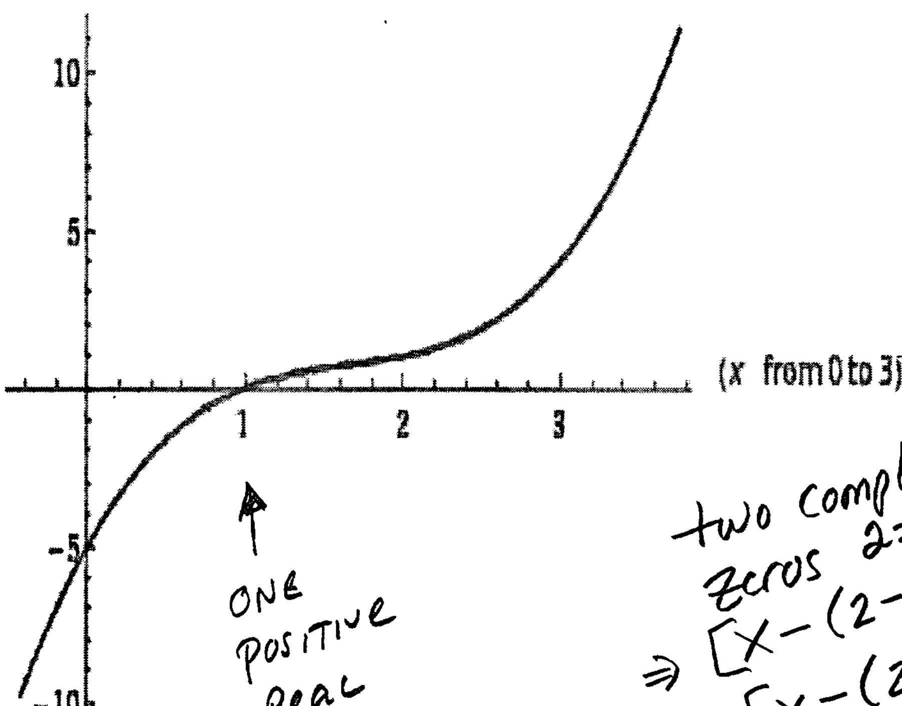
Examples Random

Input interpretation:

plot	$P(x) = x^3 - 5x^2 + 9x - 5$

Parent Function
$P(x) = x^3$
or $y = x^3$

Plots:



ONE POSITIVE REAL ZERO
$(x-1)=0$

two complex zeros $2 \pm i$
$\Rightarrow [x-(2-i)]$
AND $[x-(2+i)]$

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Ex 10-1A Find all the zeros of
pg 440 $P(x) = x^4 - 6x^3 + 15x^2 - 18x + 10$
if: $2-i$ is a zero

If $x-(2-i)$ is a zero, so is $x-(2+i)$

From the prior problem we know

$$[(x-2)-i][(x-2)+i] = (x-2)^2 - i^2$$

$$= x^2 - 4x + 5$$ ∴ this trinomial must be a factor (divide evenly) into $P(x)$

$$\begin{array}{r} x^2 - 2x + 2 \\ x^2-4x+5 \overline{\smash{\big)}\, x^4 - 6x^3 + 15x^2 - 18x + 10} \\ -(x^4 - 4x^3 + 5x^2) \\ \hline -2x^3 + 10x^2 - 18x \\ -(-2x^3 + 8x^2 - 10x) \\ \hline +2x^2 - 8x + 10 \\ +2x^2 - 8x + 10 \end{array}$$ ✓

$\therefore x^2 - 2x + 2 = 0$

$a = 1$ $\quad b^2 - 4ac$

$b = -2$ $\quad (-2)^2 - 4(1)(2)$

$c = 2$ $\quad 4 - 8 = -4 = d$

$$x = \frac{-b \pm \sqrt{d}}{2a}$$

$$x = \frac{2 \pm 2i}{2}$$

$$x = 1 \pm i$$

Zeros are $\boxed{x = 1+i,\ 1-i,\ 2-i,\ 2+i}$

WolframAlpha computational knowledge engine

Plot P(x) = x^4-6x^3+15x^2-18x+10

Examples Random

Input interpretation:

plot	$P(x) = x^4 - 6x^3 + 15x^2 - 18x + 10$

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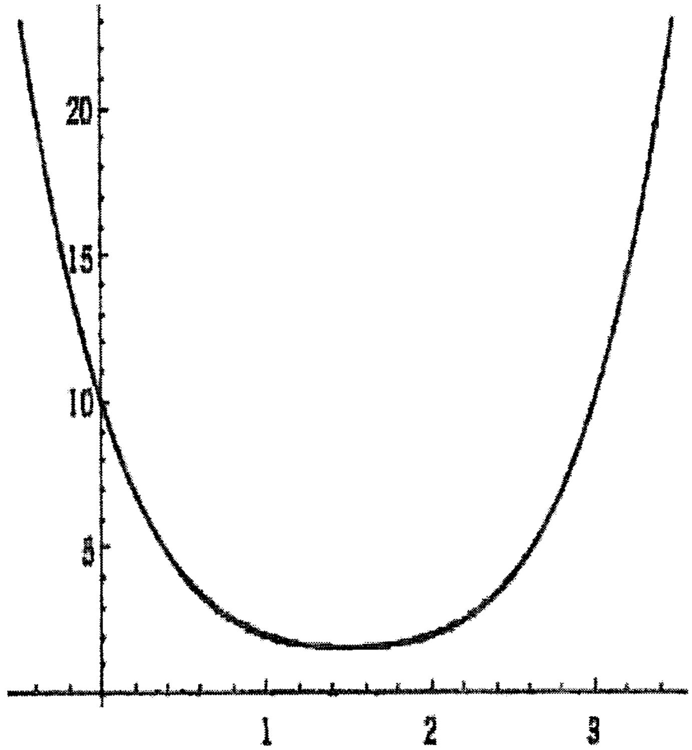

(x from 0 to 3)

∅ positive real zeros
∅ negative real zeros
4 complex zeros

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Descartes' Rule of Signs

Given: $P(x)$ with real coefficients, leading coefficient positive, constant term NOT zero, in descending order (by degree), then:

a. the number of positive real zeros is equal to the number of variations (changes) in signs — or — is less than this by a positive even number e.g. 2, 4, ...

b. the number of negative real zeros is equal to the number of variations in signs of $P(-x)$ — or — is less than this by a positive even number.

(Ex) $P(x) = x^3 - 5x^2 + 9x - 5$

$+ \quad - \quad + \quad -$

change change change 3 changes

$\therefore$ 3 or 1 positive real zeros

$P(-x) = - \quad - \quad - \quad -$

no changes $\therefore$ $\emptyset$ negative real zeros.

Ex 10-1b
pg 441

$P(x) = x^3 + 2x^2 + 3x + 1$

$+ \quad + \quad + \quad +$ No changes
∴ ∅ positive real zeros

$P(-x) = - \quad + \quad - \quad +$

change chang change ∴ 3 or 1 negative real zeros.

Ex 2
pg 442

$P(x) = 4x^4 + 2x^2 - x - 3$

$+ \quad + \quad - \quad -$

change ∴ 1 real positive zero

$P(-x) = + \quad + \quad + \quad -$

change ∴ 1 real negative zero

∴ 2 real zeros, 2 complex zeros (1 conjugate pair)

WolframAlpha computational knowledge engine

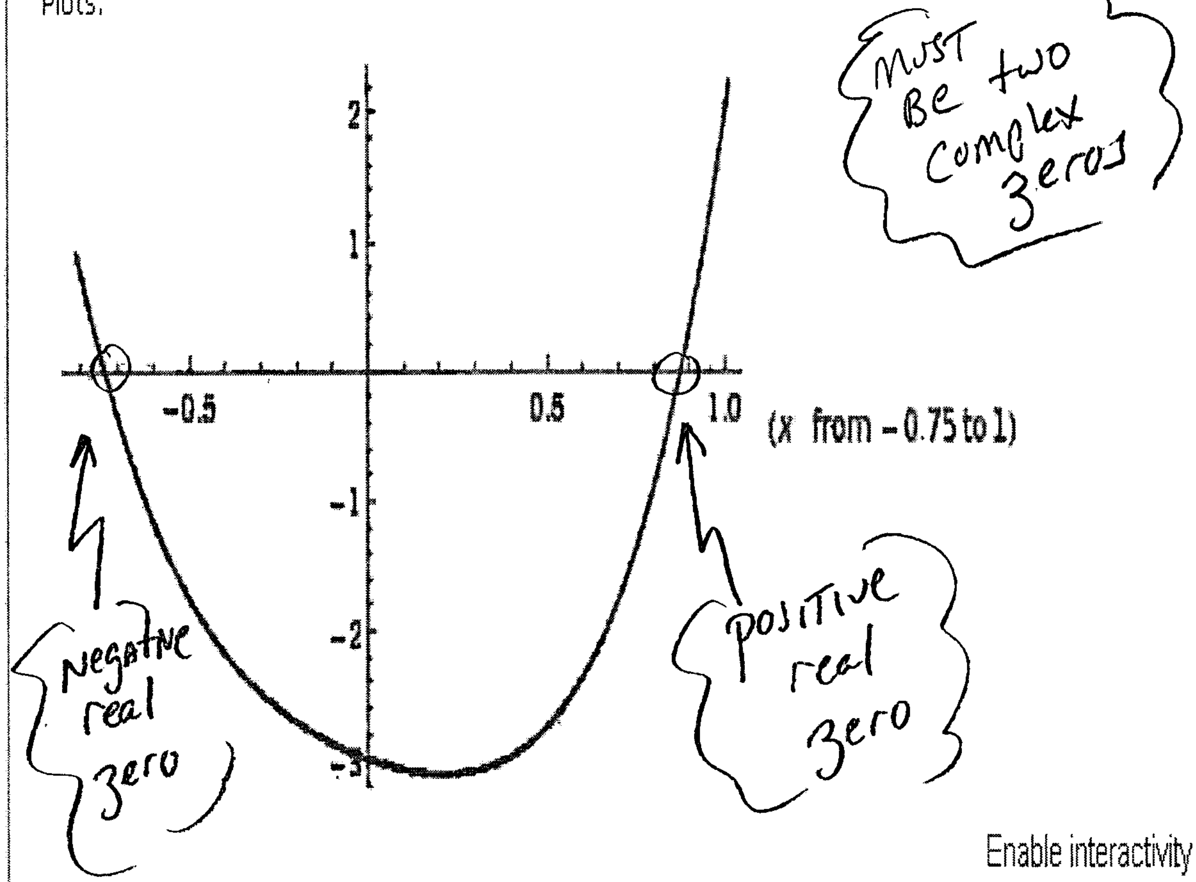
Plot P(x) = 4x^4+2x^2-x-3

Examples Random

Input interpretation:

plot	$P(x) = 4x^4 + 2x^2 - x - 3$

Plots:



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Ex 3
PG442

$P(x) = x^5 - 2x^3 + x - 1$

$+ \quad - \quad + \quad -$

CHANGE CHANGE CHANGE

$\therefore$ 3 POSITIVE or 1 POSITIVE real zeros

$P(-x) = - \quad + \quad - \quad -$

change CHANGE 2 (or zero) NEGATIVE real zeros.

TOTAL	+ real	− real	NON-real Complex (MUST BE EVEN NUMBER)
5 ✓	3	2	0
5 ✓	3	0	2
5 ✓	1	2	2
5 ✓	1	0	4

Ex 4

$P(x) = x^4 + 4x^2 + 1$

$+ \quad + \quad +$ $\emptyset$ POSITIVE real zeros

$P(-x) \quad + \quad + \quad +$ $\emptyset$ NEGATIVE real zeros

$\therefore$ $\boxed{\text{4 NON-real Complex zeros}}$