

Geometry Tues. 12-4-12 Class Notes

Only X^2 , X^1 , Numbers $\Rightarrow$
(constants)

$$aX^2 + bX^1 + c = 0$$

STANDARD FORM OF A QUADRATIC Eg.
b, c may be 0

FACTORS MULT. TO MAKE PRODUCTS

(Ex)

$\downarrow \quad \downarrow$
 $3 \cdot 8 = 24$

(Ex)

$(2x+3)(x-5)$

FACTORS

$$= 2x^2 - 10x - 15 + 3x$$

$2x^2 - 7x - 15$

PRODUCT

$a \cdot b = 0$ $a=0$ or $b=0$ or $a \text{ and } b = 0$
 Zero Product Property

$$2x^2 - 7x - 15 = 0$$

$$2x+3=0$$

$$\frac{2x}{2} = -\frac{3}{2}$$

$$\underbrace{(2x+3)}_a \cdot \underbrace{(x-5)}_b = 0$$

$x = \left\{ 5, -\frac{3}{2} \right\}$

Ex $(2x+3)(x-5) =$

$$2x^2 - 10x + 3x - 15 = 2x^2 - 7x - 15$$

- $2x^2 - 7x - 15 = 0$
 - Pot in SF
 - Read a, b, c
 - Find magic numbers
 - FBG
- sum = b = -7
 prod = ac = -30
- Magic numbers: 1, 30; 2, 15; 3, 10; 5, 6

FBG

$$(2x^2 + 3x) + (-10x - 15) = 0$$

$$x(2x+3) - 5(2x+3) = 0$$

Ex $\begin{matrix} x & -5 \\ \hline (x-5) \end{matrix}$ Why it works

$$(2x+3)(x-5) = 0 \quad \text{Solution Set}$$

$2(5)^2 - 7(5) - 15 \stackrel{?}{=} 0$
 $x = \left\{ 5, -\frac{3}{2} \right\}$

$2\left(-\frac{3}{2}\right)^2 - 7\left(-\frac{3}{2}\right) - 15 \stackrel{?}{=} 0$
 $2\left(\frac{9}{4}\right) + \frac{21}{2} - 15 \stackrel{?}{=} 0$
 $\frac{9}{2} + \frac{21}{2} - 15 \stackrel{?}{=} 0 \checkmark$

$$\textcircled{1} \quad \begin{array}{ccc} x^2 + 2 & = & -3x \\ +3x & & +3x \end{array}$$

$$ax^2 + bx + c = 0$$

$$x^2 + 3x + 2 = 0$$

$$\text{sum} = b = 3$$

$$\text{prod} = a \cdot c = 2$$

$$\begin{pmatrix} +1 \\ +2 \end{pmatrix}$$

Magic Numbers

$$+1x + 2x$$

egw

$$(x^2 + 1x) + (2x + 2) = 0$$

$$x(\underline{x+1}) + 2(\underline{x+1}) = 0$$

$$\underline{(x+1)}(\underline{x+2}) = 0$$

$$\boxed{x = \{-1, -2\}}$$

zpf

a = 0

b

= 0

✓ ✓

Start
cut

$$\text{If } a = +1 \quad \text{MN}^3 \Rightarrow +1, +2$$

FBG

not
needed

$$(x+1)(x+2) = 0$$

$$\textcircled{7.} \quad 6 = -b^2 + 5b$$

$$\textcircled{a = +1}$$

 SHORTCUT

$$b^2 - 5b + 6 = 0$$

$$\text{Sum} = b = -5$$

$$\text{prod} = ac = 6$$

$$\begin{array}{cc} \diagup & \diagdown \\ -2 & -3 \end{array}$$

$$(\cancel{b} - 2)(b - 3) = 0 \quad \boxed{b = \{2, 3\}}$$

$$\textcircled{25} \quad -4b^2 = -4$$

$$4b^2 \text{ } \textcircled{} - 4 = 0$$

$$\begin{array}{c} \uparrow \\ +0b^1 \\ \hline \end{array}$$

$$a = 4$$

$$b^2 - 4ac$$

$$b = 0$$

$$(0)^2 - 4(4)(-4)$$

$$c = -4$$

$$\boxed{+64 = d}$$

two rational
real solutions

$$2x^2 - 7x - 15$$

$$a = 2$$

$$b^2 - 4ac$$

$$b = -7$$

$$(-7)^2 - 4(2)(-15)$$

$$c = -15$$

$$49 + 120 = \boxed{169 = d}$$

^d
discriminant
 $b^2 - 4ac$

d is a perfect square — you can factor the quadratic (i.e., the magic numbers exist)

If d is negative, there are NO real solutions to the quadratic

WS
 (2) $a^2 + 2a = 15$
 $-15 \quad -15$

SF
 $ax^2 + bx + c = 0$

$$a^2 + 2a - 15 = 0$$

$\uparrow \quad \uparrow \quad \uparrow$
 $a=1 \quad b=2 \quad c=-15$

$$\text{sum} = b = 2$$

$$\text{prod} = ac = -15$$

$$-3a + 5c$$

$$\begin{array}{r} 1, 15 \\ 3, 5 \end{array}$$

FBG $(a^2 - 3a) + (5a - 15) = 0$

$$a(\underline{a-3}) + 5(\underline{a-3}) = 0$$

$$(\underline{a-3})(a+5) = 0$$

$$a = \{ \underline{3}, \underline{5} \}$$

CK $(\quad)^2 + 2(\quad) \stackrel{?}{=} 15$

Find d

$$a = 1$$

$$b = 2$$

$$c = -15$$

$$b^2 - 4ac$$

$$(2)^2 - 4(1)(-15)$$

$$4 + 60 = \boxed{64 = d}$$

✓✓ Perf sq.

2 REAL, RATIONAL SOLUTIONS

WS (24) $24N = -16 - 5N^2$ Std Form

$+5N^2 + 16 \quad +16 \quad +5N^2$

$$5N^2 + 24N + 16 = 0$$

Sum = b = 24 10, 8
 prod = ac = 80 80, 1
 20, 4

$+20 + 4$

$$(5N^2 + 20N) + (4N + 16) = 0$$

$$5N(\underline{N+4}) + 4(\underline{N+4}) = 0$$

$$(\underline{N+4})(5N+4) = 0$$

$$N = \left\{ -4, -\frac{4}{5} \right\}$$

$$2x^2 - 7x - 15$$

$$a = 2 \quad b^2 - 4ac$$

$$b = -7 \quad (-7)^2 - 4(2)(-15)$$

$$c = -15 \quad 49 + 120 = \boxed{169 = d}$$

↑

Perf sq

⇒ Factorable

⇓

Magic numbers

EXIST

Find Discriminant

$$(11) \quad k^2 + 7k = -10 \quad \text{SF} \quad ax^2 + bx + c = 0$$

$$k^2 + 7k + 10 = 0$$

$$\text{sum} = b = 7$$

$$\text{prod} = ac = 10$$

$$+2 + 5 \Rightarrow (k+2)(k+5) = 0$$

$$(k^2 + 2k) + (5k + 10) = 0$$

$$k(\underline{k+2}) + 5(\underline{k+2}) = 0$$

$$(\underline{k+2})(k+5) = 0$$

$$k = \{-5, -2\}$$

$$a = 1$$

$$b^2 - 4ac$$

$$b = 7$$

$$(7)^2 - 4(1)(10)$$

$$c = 10$$

$$49 - 40 = 9 = d$$

Perf 58.

(23)

$$\downarrow 3x^2 - 10 = x$$

$$3x^2 - x - 10 = 0$$

$$\text{sum} = -1$$

$$\text{prod} = -30$$

$$+5 -6$$

$$(3x^2 - 6x) + (5x - 10) = 0$$

$$3x(\underline{x-2}) + 5(\underline{x-2}) = 0$$

$$(\underline{x-2})(3x+5) = 0$$

$$x = \left\{ 2, -\frac{5}{3} \right\}$$

? ?