Algebra 2 | Monday 10-29-12 | Class Notes

Ch. 4-4 Properties of Logarithms ?

Recall: Multiplication Rule for Exponents

?What is a logarithm?

$$MR \quad a^m \cdot a^n = a^{m+n}$$

When multiplying powers with same base, add exponents.

So what might adding logarithms involve?

$$\boxed{\log_b(MN) = \log_b M + \log_b N}$$

Multiplication Rule for Logs

Ex $\log_4 (2)(32) = \log_4 2 + \log_4 32$

→ this is called expanding the logarithm.

Ex $\log_5(625) + \log_5(25) = \log_5(625 \cdot 25)$

→ this is called condensing the logarithm

Ex 1 A
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$\log_4 2 + \log_4 32$

$\downarrow \quad \downarrow$

$? \quad ?$

$= \log_4 (2 \cdot 32) = \log_4 64$

$= 4^x = 64$

$x = 3$

$\therefore \boxed{\log_4 2 + \log_4 32 = 3}$

Ex 1
(CONT)

$\log_5 625 + \log_5 25$

$= \log_5 (\underset{5^4}{625} \cdot \underset{5^2}{25}) = \log_5 (5^6)$

$5^4 \cdot 5^2 \qquad 5^x = 5^6$

$x = 6$

$\therefore \boxed{\log_5 625 + \log_5 25 = 6}$

$4 + 2 = 6$ ✓

Ex

$\log_{\frac{1}{3}}(27) + \log_{\frac{1}{3}}\left(\frac{1}{9}\right)$

$= \log_{\frac{1}{3}}\left(27 \cdot \frac{1}{9}\right) = \log_{\frac{1}{3}}(3)$

$\left(\frac{1}{3}\right)^x = 3$

$\boxed{x = -1}$

$\therefore \log_{\frac{1}{3}}(27) + \log_{\frac{1}{3}}\left(\frac{1}{9}\right) = -1$

Recall:

Division Rule of Exponents

$$\frac{a^m}{a^N} = a^{m-N}$$

$$\boxed{\log_b\left(\frac{m}{N}\right) = \log_b m - \log_b N}$$

Division Rule of Logarithms

EX2
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$$\log_2(32) - \log_2(4) = ?$$

Condense $\Rightarrow$ $\log_2\left(\frac{32}{4}\right) = \log_2(8)$

$$\Rightarrow 2^x = 8$$

$$x = 3$$

$$\therefore \boxed{\log_2(32) - \log_2(4) = 3}$$

Finally, Recall the Power-Power Rule of Exponent

$$(a^m)^N = a^{m \cdot N}$$

$$\boxed{\log_b(a^m) = m \log_b a}$$

Power-Power Rule for Logarithms.

This one is sneaky as it is much more useful than it looks – it is used extensively in higher level maths.

Ex $\log_3 81^2$

$= 2\underbrace{\log_3 81}$

$= 2(4)$

$= \boxed{8}$

Since $3^x = 81$
$x = 4$
$\log_3 81 = 4$

More Examples — Pg 257

Ex $\log_5\left(\frac{1}{5}\right)^3$

Ex $\log 10^4$

Ex $\log_5 25^2$

Ex $\log_2\left(\frac{1}{2}\right)^5$

Take a relatively single expression like $\log_4 8$ and evaluate it using a scientific calculator — you can even use a TI30XS Multiview!

No luck?

Only 2 bases are built into most calculators — base 10 and base 2.71828...

Common Logarithms	NATURAL Logarithms
base 10	base e
	$e \approx 2.718281828\ldots$
$\boxed{\text{Log}}$	$\boxed{\text{ln}}$
	"logarithmus naturalis"

Change of Base Formula
(can use any base but to use you calculator use base 10 or base e)

Ex

$$\log_4 8 \underset{\text{bottom}}{\overset{\text{top}}{=}} \frac{\log_{10} 8}{\log_{10} 4} \cong \frac{.903089987}{.60205991} = \boxed{1.5 \text{ or } \frac{3}{2}}$$

OLD BASE 4

NEW BASE 10

Check $4^{\frac{3}{2}} \stackrel{?}{=} 8$

$\left(\sqrt[2]{4}\right)^3 \stackrel{?}{=} 8$ ✓

$4 \wedge 1.5 \cong 8$

Change of Base Formula - Pg 258

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

Try $\log_4 8$ with $\boxed{\ln}$ instead of $\boxed{\log}$