Algebra 2 | Tuesday 2-5-13 | Class Notes

Ch. 9-4 Geometric Sequences and Series

geometric sequence — the ratio of successive terms is a constant called the common ratio, r

$(r \neq 1)$

(Ex) 128, 64, 32, 16, ...

$\frac{64}{128} = \frac{1}{2} = r$

$\therefore 32 \cdot \frac{1}{2} = 16$ ✓

$64 \cdot \frac{1}{2} = 32$ ✓

The N^{th} term of a geometric sequence is $a_N = a_1 r^{N-1}$

(Ex) term 5 of sequence above

is $a_N = a_1 r^{N-1}$

$a_5 = 128\left(\frac{1}{2}\right)^{5-1}$

$= 128\left(\frac{1}{2}\right)^4$

$= 128\left(\frac{1}{16}\right)$

$a_5 = 8$

$\frac{128}{16} = \frac{64}{8} = 8$

EX2 Pg 655 Find 9th term of geometric sequence $-5, 10, -20, 40, -80, \ldots$

$r = \frac{10}{-5} = -2$

$a_1 = -5$

$a_N = a_1 r^{N-1}$

$a_9 = (-5)(-2)^{9-1}$

$a_9 = (-5)(-2)^8$

$\Downarrow$ positive

$a_9 = (-5)(256)$

$a_9 = -1280$

Finding N^{th} term given 2 terms

(Ex3) PG656 Find 10th term of geometric sequence $a_5 = 96$, $a_7 = 384$

Find r: $a_N = a_1 r^{N-1}$

$96, \_\_, 384$ (a_1, a_3)

$a_3 = a_1 r^{3-1}$

$384 = 96(r)^2$

$\frac{384}{96} = r^2$

$\frac{128}{32} = r^2$

$\frac{64}{16} = r^2$

$4 = r^2$ $\therefore$ $r = \pm 2$ *

Using +2

Find a_1: $a_7 = a_1 r^{N-1}$

$384 = a_1 (2)^{7-1}$

$\frac{384}{2^6} = a_1 = \frac{384}{64} = \frac{192}{32} = \frac{96}{16} = 6$

$a_1 = 6$

$\therefore a_{10} = a_1 r^{N-1}$

$a_{10} = 6(2)^{10-1}$

$= 6(2)^9 = 6(512)$

$a_{10} = 3072$

* See Book for $r = -2 \Rightarrow -3072$

The Partial Sum S_N of the first N terms of a geometric series

$$S_N = a_1\left(\frac{1-r^N}{1-r}\right) \quad r \neq 1$$

Pg 657

(EX5) Pg 657 Find S_7 for $3, -6, +12, -24$

$r = \frac{-6}{3} = -2$

$N = 7$

$a_1 = 3$

$$S_N = a_1\left(\frac{1-r^N}{1-r}\right) = 3\left(\frac{1-(-2)^7}{1-(-2)}\right)$$

$$= 3\left(\frac{1-(-128)}{1+2}\right)$$

$$= 3\left(\frac{129}{3}\right)$$

$$S_7 = 129$$

geometric mean — if a, b are positive terms of a geometric sequence with only 1 term between them, the geometric mean is defined as $\sqrt{ab}$

(Ex) $5, 10, 20, 40 \ldots$

geometric mean of $10, 40$ is $\sqrt{10 \cdot 40} = \sqrt{400} = 20$

(Ex) $\frac{1}{2}, \frac{1}{32}$

$$\text{Geo. mean} = \sqrt{\tfrac{1}{2} \cdot \tfrac{1}{32}} = \sqrt{\tfrac{1}{64}} = \tfrac{1}{8}$$

(Ex) Find geo. mean of 16 and 25

$$= \sqrt{16 \cdot 25} = \sqrt{16}\sqrt{25} = 4 \cdot 5 = 20$$

Homework Review (Arithm. Seq)

(13) Find a_9 $\quad a_3 = -5 \quad a_6 = -11$

$a_N = a_1 + (N-1)d$

a_1 $\quad a_4$

Find d:

$-11 = -5 + (4-1)d$

$-11 = -5 + 3d$

$-6 = 3d \therefore d = -2$ *

Use $a_3 = -5$ to find a_1

$-5 = a_1 + (3-1)(-2)$

$-5 = a_1 + -4$

$-1 = a_1$ *

$\therefore a_9 = a_1 + (9-1)(-2)$

$a_9 = -1 + (8)(-2)$

$a_9 = -17$ ✓

(19) S_{18} for $3.2 + 2.9 + 2.6 + 2.3 \ldots$

a_1

$d = -0.3$

$$S_N = \frac{N}{2}(a_1 + a_N)$$

$$a_N = a_1 + (N-1)\,d$$

$$a_{18} = 3.2 + (17)(-.3)$$

$$= 3.2 - 5.1$$

$$a_{18} = -1.9$$

$$5.1 - 3.2 = 1.9$$

$$S_{18} = \frac{18}{2}(3.2 - 1.9)$$

$$= 9(+1.3)$$

$$S_{18} = 11.7$$

$$3.2 - 1.9 = +1.3$$