Algebra 2 | Monday 2-4-13 | Class Notes

Sequence An ordered set of numbers, may be finite (ends) or infinite. (never ends)

Sequences are functions, notation is custom for sequences

N = independent variable
i.e. the term number

a_N = the value of term N

(Ex)

N	a_N
1	1
2	1
3	2
4	3
5	5

term 3 $\Rightarrow N=3$
$a_3 = 2$
or $(3, 2)$
N, a_N

(Ex) $4, 6, 18, 109, 132, 211$

$N=1$ $N=2$ $N=3$ $N=4$ $N=5$ $N=6$

a_1 a_2 a_3 a_4 a_5 a_6

Series — the SUM of A CERTAIN number of terms of a sequence. If the sum does not include all terms in the sequence it is called A PARTIAL SUM, S_N

(EX) Sequence 1, 2, 3, 4

Series $1+2+3+4$

SUMMATION NOTATION

Sigma $\sum$ (Greek Letter) ⇓ "" means "SUM"

(EX) $\sum_{k=1}^{5} 2k = 2(1)+2(2)+2(3)...+2(4)+2(5)$

K is the INDEX, the starting VALUE is on the bottom of the $\sum$ notation, the ending VALUE is on the top of the $\sum$

The "RULE" for finding each term in the series is on the side $\sum$ rule

(Ex) Pg 634 Write in summation notation

$$3+6+9+12+15$$

↑ term 1, $K=1$ ↑ term 5, $K=5$

RULE is $3(K)$, from 1 to 5

$$\therefore \sum_{K=1}^{5} 3K = 3+6+9+12+15$$

Ch. 9-1 Intro. to Sequences
Ch 9-2 Series and Summation Notation

Ch. 9-3 Arithmetic Sequences and Series

Arithmetic Sequence — EACH term differs by the SAME number, ADD or subtract, CALLED the common difference, d

(EX) 8, 12, 16, 20, ...
$d = +4$

(EX) 6, 4, 2, 0, -2, ...
$d = -2$

(EX) 15, 20, 24, 29,
NOT AN ARITHMETIC Sequence

Since EACH term, a_N is the sum of the previous term, a_{N-1} AND the common difference, $a_N = a_{N-1} + d$

If you know the 1st term, a_1, then the N^{th} term can be found from

* $\boxed{a_N = a_1 + (N-1)d}$

(Ex 2) Pg 644 Find 10th term of arithmetic sequence 32, 25, 18, 11, 4...

$N=10$

term 1 $= a_1$ $\quad d = -7$

$\therefore a_N = a_1 + (N-1)d$

$a_{10} = 32 + (10-1)(-7)$

$a_{10} = 32 + 9(-7)$

$a_{10} = 32 - 63$

$\boxed{a_{10} = -31}$

(Ex 3) Pg 645 Find missing terms in arithmetic sequence: $11, \_, \_, \_, -17$

$N=5$

$a_N = a_1 + (N-1)d$

$-17 = 11 + (5-1)d$

$-17 = 11 + (4d)$

$-11 \quad -11$

$-28 = 4d \quad \therefore \boxed{d = -7}$

$\therefore a_2 = 11 - 7 = \boxed{4}$

$a_3 = 4 - 7 = \boxed{-3}$

$a_4 = -3 - 7 = \boxed{-10}$

(EX4) Pg645 Find N^{th} term given 2 terms (in an Arithmetic sequence)

Given: $a_9 = 120$, $a_{14} = 195$

Find: 6 term, i.e. a_6

Find d $\quad a_N = a_1 + (N-1)d$

$a_{14} = a_9 + (6-1)d$

$195 = 120 + 5d$

$-120 \quad -120$

$75 = 5d \quad \therefore \boxed{d = 15}$

Find a_1 $\quad a_N = a_1 + (N-1)d$

$a_9 = a_1 + (9-1)(15)$

$120 = a_1 + 8(15)$

$120 = a_1 + 120$

$\boxed{0 = a_1}$

$\therefore a_6 \Rightarrow a_N = a_1 + (N-1)d$

$a_6 = 0 + (6-1)(15)$

$\boxed{a_6 = 75}$

ARITHMETIC SERIES — the sum of the terms of an arithmetic sequence

(EX) Find the sum of the first 100 terms of $1+2+3+4+5...$

Note: $a_1 + a_{100} = 1 + 100 = 101$

$a_2 + a_{99} = 2 + 99 = 101$

How many pairs are there? $\frac{N}{2} = \frac{100}{2} = 50$

So $S_{100} = \frac{N}{2}(a_1 + a_N)$

$= \frac{100}{2}(101)$

$S_{100} = 50(101) = 5050$

Pg 546

$S_N = \left(\frac{N}{2}\right)(a_1 + a_N)$ or $N\left(\frac{a_1 + a_N}{2}\right)$

(PARTIAL SUM)

(Ex 5) Find the indicated sum for each arithmetic series
Pg 646

(A) S_{15} for $25 + 12 + (-1) + (-14) \ldots$

$$S_N = \left(\frac{N}{2}\right)(a_1 + a_N)$$

$$S_{15} = \frac{15}{2}(25 + a_N)$$

$a_N = a_1 + (N-1)d$ $\quad d = -13$

$a_{15} = 25 + (15-1)(-13)$

$= 25 + (14)(-13)$

$= 25 - 182$

$a_{15} = -157$

$$\therefore S_{15} = \frac{15}{2}(25 + (-157))$$

$$= \frac{15}{2}(-132)$$

$$= -15(66)$$

$$S_{15} = -990$$

$$\begin{array}{r} 14 \\ \times 13 \\ \hline 42 \\ 14 \\ \hline 182 \end{array}$$

$$\begin{array}{r} 66 \\ \times 15 \\ \hline 330 \\ 66 \\ \hline 990 \end{array}$$

(Ex B) $\sum_{K=1}^{12} (3+4K)$

$a_1 = 3+4(1) = 7$

$a_{12} = 3+4(12) = 51$

$S_N = \frac{N}{2}(a_1 + a_N)$

$S_{12} = \frac{12}{2}(7+51)$

$= \frac{12}{2}(58)$

$S_N = 348$

$S_{12} = 348$