ALGEBRA II WITH TRIGONOMETRY

Algebra II With Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II With Trigonometry course.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

Use complex numbers in polynomial identities and equations. *(Polynomials with real coefficients.)*

3. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

4. (+) Extend polynomial identities to the complex numbers.
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. [N-CN8]

5. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. *(Polynomial and rational.)*

6. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
   a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
      Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

7. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
     Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

8. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
     Example: Calculate mortgage payments.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Beyond quadratic.)*

9. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Understand the relationship between zeros and factors of polynomials.

10. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

11. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Use polynomial identities to solve problems.

12. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
     Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Rewrite rational expressions. *(Linear and quadratic denominators.)*

14. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

15. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. [A-APR7]

Creating Equations*

Create equations that describe numbers or relationships. *(Equations using all available types of expressions, including simple root functions.)*

16. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

17. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

18. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
     Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

19. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
     Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning, and explain the reasoning. *(Simple rational and radical.)*

20. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Represent and solve equations and inequalities graphically. *(Combine polynomial, rational, radical, absolute value, and exponential functions.)*

21. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. *(Emphasize selection of appropriate models.)*

22. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* [F-IF4]

23. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

24. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Analyze functions using different representations. *(Focus on using key features to guide selection of appropriate type of model function.)*

25. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

26. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

27. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. *(Include all types of functions studied.)*

28. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Combine standard function types using arithmetic operations. [F-BF1b]
        Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. *(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)*

29. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

30. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
    Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. *(Logarithms as solutions for exponentials.)*

31.　For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

32.　Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. [F-TF1]

33.　Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. [F-TF2]

34.　Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Model periodic phenomena with trigonometric functions.

35.　Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* [F-TF5]

Prove and apply trigonometric identities.

36.　Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. [F-TF8]

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

37.　Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

38. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

39. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

40. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

41. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

42. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

43. Evaluate reports based on data. [S-IC6]

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (*Include more complex situations.*)

44. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

45. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]