BE - Geometry 1

Wednesday 2-29-12

Ch. 11-3 - Areas of Regular Polygons and Circles

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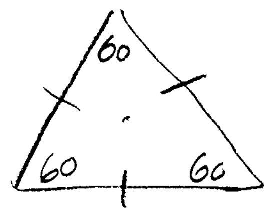
Regular Polygon — A convex polygon (∠s less than 180°) with all sides congruent AND all interior ∠s congruent

All sides $\cong$ AND All ∠s $\cong$

EXAMPLES

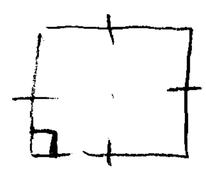
Interior Angle

Regular △



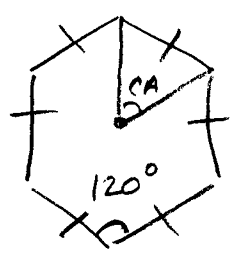
Equilateral △
$180°/3$

Regular 



Square
$360°/4$

Regular 



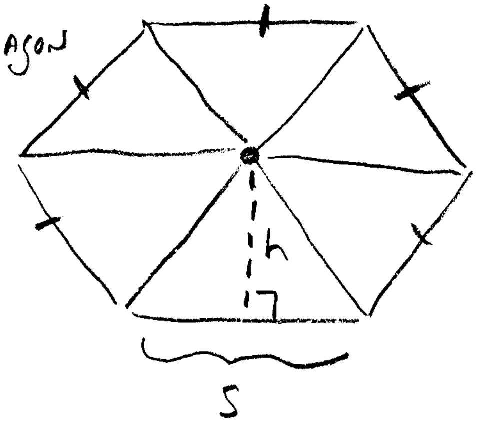
hexagon
$720°/6$

Central Angle $\Rightarrow \dfrac{360}{\text{Number of sides}}$

(Ex) hexagon $\Rightarrow \dfrac{360}{6} = 60°$

Any regular polygon with N sides can be split into N congruent triangles:

Ex Hexagon



The area of each Δ is $\frac{1}{2}sh$

So the area ⬡ is $6 \cdot \frac{1}{2}sh$

or $\frac{1}{2} \cdot 6sh$

but the Perimeter, $P = 6s$

$\therefore \boxed{A = \frac{1}{2}Ph}$

h = the perpendicular drawn from the center of a regular polygon to a side of the polygon

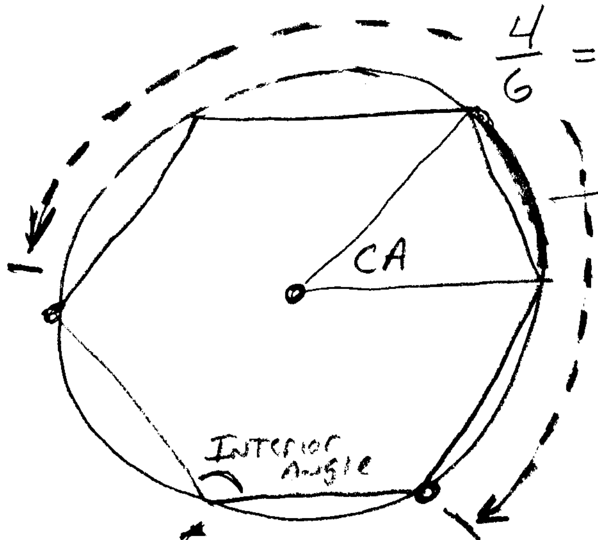
It is called the apothem, a

A circumscribed circle outside of a regular polygon shows the properties of the interior and central angles.

$\boxed{\text{Intercepted Arc of Interior } \angle}$

$\frac{4}{6} = \frac{2}{3}$ of $360 = 240$

$\boxed{\text{Central } \angle}$



$\frac{1}{6} \cdot 360 = 60^\circ$

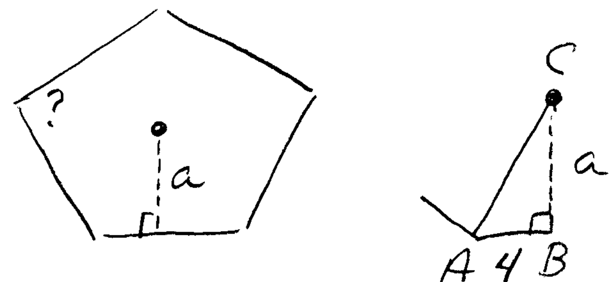
$\frac{1}{2}$ of $240 = 120^\circ$

$\boxed{\text{Central Angle} = \frac{1}{2} \text{ Interior Angle}}$

$\therefore$ if P is the perimeter
And a is the apothem

$$\boxed{\text{Area of A Regular Polygon} = \frac{1}{2}Pa}$$

Ex 1 Pg 611 Find the Area of a Regular pentagon with a perimeter of 40 cm. $\Rightarrow$ Each side $= \frac{40}{5} = 8$ cm



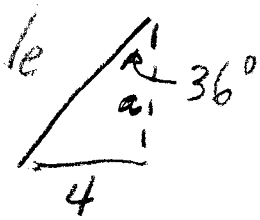
measure of each angle is $\frac{360+180}{5} = \frac{540}{5} = 108^\circ \therefore m\angle A = 54^\circ$

$\tan A = \frac{a}{4} \therefore a = 4\tan 54^\circ$

$a = 4(1.3764) = 5.5056$

$\therefore \text{Area}_{\pentagon} = \frac{1}{2}(40)5.5056 \cong \boxed{110.1 \text{ cm}^2}$

Book used the $\frac{\text{Central angle}}{2}$



$\tan 36^\circ = \frac{4}{a}$

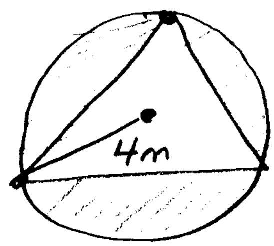
Area of Circle $\Rightarrow$ * $\boxed{A_{\odot} = \pi r^2}$

Since $d = 2r \Rightarrow r = \frac{d}{2}$ $\therefore A_{\odot} = \pi\left(\frac{d}{2}\right)^2$

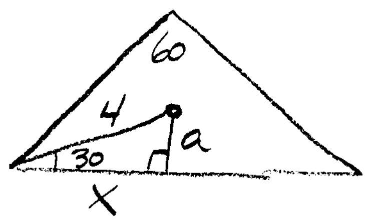
$$\boxed{A_{\odot} = \frac{\pi d^2}{4}}$$

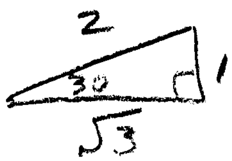
EX3 Pg 612 Equilateral $\triangle$ inscribed in circle. Find Area of shaded region.

Area $\odot = \pi(4)^2 = 16\pi$ meters^2



$A_{\triangle} \Rightarrow$

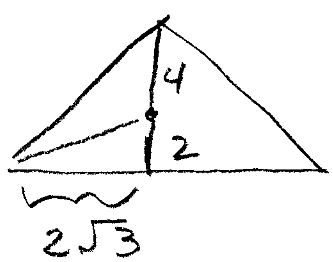




$\therefore a = 2$ $\therefore x = 2\sqrt{3}$

$\therefore$ 1 side of $\triangle = 2(2\sqrt{3}) = 4\sqrt{3}$

base $= 2\sqrt{3}$



$A_{\triangle} = \frac{1}{2}(4\sqrt{3})6 = 12\sqrt{3}$

$\therefore A_{SHADED} = 16\pi - 12\sqrt{3}$

$\cong 50.27 - 20.78$

Homework: Pg 613 # 3, 6, 14, 19

$\boxed{A \cong 29.49\ m^2}$