

BE Alg. 2

Tuesday 11-1-11

ACT
BARRONS

①. WHAT IS THE MATRIX PRODUCT?

$$\begin{bmatrix} x & 2x & 4x \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

Ans)

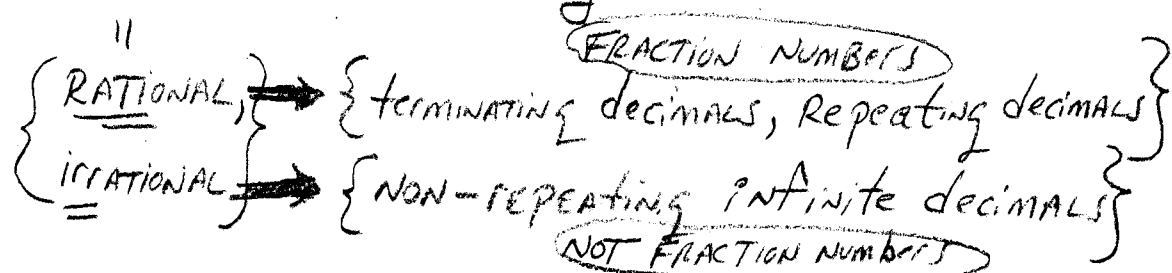
$1 \times 3 \cdot 3 \times 1 \checkmark$ OK
product matrix is 1×1 $3=3 \checkmark$

$$-2 \ 0 \ 5$$

$$\begin{bmatrix} x & 2x & 4x \end{bmatrix} \rightarrow \begin{bmatrix} -2x + 0 + 20x \end{bmatrix}$$

$$\Rightarrow \boxed{18x} \leftarrow \text{ANS}$$

Using Real Numbers Only:



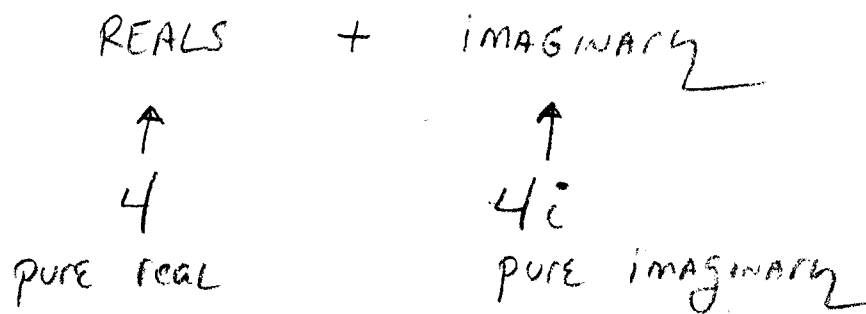
Why does $\sqrt{-16}$ NOT EXIST?

$$(-4)(-4) \neq -16 \quad \text{AND} \quad (4)(4) \neq -16$$

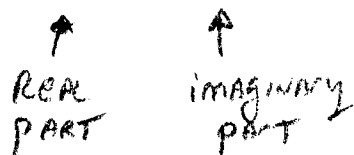
$$\text{Since } \sqrt{-16} = \sqrt{-1} \sqrt{16} \quad \boxed{\text{DEFINE: } \sqrt{-1} = i}$$

$$\therefore \sqrt{-16} = i 4 = 4i$$

imaginary number



$$4 + 4i$$



$$\boxed{\text{COMPLEX NUMBERS}} = \{\text{REALS, IMAGINARY}\}$$

$$\textcircled{\text{EX}} \quad \sqrt{-25} = 5i$$

$$\sqrt{-18} = \sqrt{9}\sqrt{2}i = 3\sqrt{2}i = \boxed{3i\sqrt{2}}$$

$$\sqrt{-1} = i$$

WHY preferred?

Multiplying Imaginary Numbers:

EX 1
pg 270

⇒ Ch. 5-9 Complex Numbers

$$\textcircled{\text{A.}} \quad -2i \cdot 7i = -14i^2 = -14(-1) = \boxed{14}$$

WHY?

*

$$\begin{aligned} i &= \sqrt{-1} = i \\ i^2 &= (\sqrt{-1})^2 = -1 \\ i^3 &= i^2 i = -i \\ i^4 &= i^2 i^2 = +1 \end{aligned}$$

Know
This!

$$\begin{aligned} i^4 &= (i^2)^2 \\ &= (-1)^2 = 1 \end{aligned}$$

$$\begin{aligned} \textcircled{\text{B}} \quad \sqrt{-10} \cdot \sqrt{-15} &= (i\sqrt{10})(i\sqrt{15}) = i^2 \sqrt{150} \\ &= (-1)\sqrt{25}\sqrt{6} \\ &= -1 \cdot 5 \cdot \sqrt{6} \\ &= \boxed{-5\sqrt{6}} \end{aligned}$$

$$\Rightarrow \sqrt{(-10) \cdot (-15)}$$

$$\sqrt{-1} \cdot \sqrt{10} \cdot \sqrt{-1} \cdot \sqrt{15}$$

$$\text{If } \sqrt{-1} \text{ EXISTS} \Rightarrow +1 \cdot \sqrt{150} = -5\sqrt{6}$$

Writing Real Numbers as Complex AND
Vice versa...

$$12 = 12 + 0i$$

$$8i = 0 + 8i$$

$$6 + 5i = 6 + 5i$$

powers of i

Ex 3
Pg 270

$$i^{45} = ?$$

$$i^1 \cdot i^{44} = i (i^2)^{22}$$

$$i (-1)^{22} = \boxed{i}$$

Ex

$$i^{58} = ?$$

$$i^1 i^{57}$$

$$i [i^1 \cdot i^{56}]$$

$$i [i \cdot (i^2)^{28}]$$

$$i [i \cdot (-1)^{28}]$$

$$i [i] = i^2 = \boxed{-1}$$

SHORTCUT

$$i^N = i^R$$

$R = \text{Remainder of } \frac{N}{4}$

$$i^{58} \Rightarrow \begin{array}{r} 14 \text{ R } 2 \\ 4 \overline{) 58} \\ \underline{4} \\ 18 \\ \underline{16} \\ 2 \end{array}$$

$$i^{58} = i^2 = \boxed{-1} \checkmark$$

SOLVING AND CHECKING SOLUTIONS TO EQUATIONS WHEN THE SOLUTIONS ARE COMPLEX:

EX 4
pg 271 SOLVE $3x^2 + 48 = 0$

$$3x^2 = -48$$

$$x^2 = -16$$

$$x = \pm \sqrt{-16} = \boxed{\pm 4i}$$

CK
4i $3(\quad)^2 + 48 \stackrel{?}{=} 0$

$$3(4i)^2 + 48 \stackrel{?}{=} 0$$

$$3 \cdot 16 \cdot i^2 + 48 \stackrel{?}{=} 0$$

$$48(-1) + 48 = 0 \quad \checkmark$$

CK
-4i $3(-4i)^2 + 48 \stackrel{?}{=} 0$

$$3 \cdot 16 i^2 + 48 \stackrel{?}{=} 0 \quad \checkmark \text{ SINCE SAME AS ABOVE}$$

OR
(FINISH) $48 i^2 + 48 \stackrel{?}{=} 0$

$$48(-1) + 48 = 0 \quad \checkmark$$

Property of Equality:

Two complex numbers ARE EQUAL iff the real parts ARE EQUAL and the imaginary parts ARE EQUAL.

EX $2x + 5i = 2x + 5i$

EX5
Pg 271 IF $2x - 3 + (y - 4)i = 3 + 2i$

Then $\underbrace{2x - 3}_{\text{Real}} + \underbrace{(y - 4)i}_{\text{imag.}} = \underbrace{3}_{\text{Real}} + \underbrace{2i}_{\text{imag.}}$

$$2x - 3 = 3 \quad \therefore 2x = 6 \quad \boxed{x = 3}$$

$$\text{AND } y - 4 = 2 \quad \therefore \boxed{y = 6}$$

✓o ADD AND SUBTRACT complex numbers

⇒ ADD REAL PARTS, ADD complex parts

EX6
Pg 272 (A) $(6 - 4i) + (1 + 3i) = \boxed{7 - i}$

(B) $(3 - 2i) - (5 - 4i) = \boxed{-2 + 2i}$

Multiplying Complex Numbers

⇒ Like Multiplying Binomials = "FOIL"

EX 7
Pg 273

$$(1+3i)(7-5i)$$

$$7 - 5i + 21i - 15i^2$$

$$7 + 16i - 15(-1)$$

$$7 + 16i + 15 = \boxed{22 + 16i}$$

NOTE: In division ⇒ don't leave i in bottom.

$$\textcircled{\text{EX}} \quad \frac{1}{4i} \cdot \frac{4i}{4i} = \frac{4i}{-16} = \frac{-i}{4}$$

$$\textcircled{\text{EX}} \quad \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{4-9(-1)} = \frac{2-3i}{13}$$

Homework: Pg. 273 # 4-11

Pg 274 # 48-51