INTRODUCTION

THE 3 "LOGARITHM RULES"

Each Log Rule is related to an Exponent Rule.
Why? Because a logarithm is an...
EXPONENT!

Multiplication Rule (for exponents) $2^3 \cdot 2^4 = 2^7$

"When multiplying powers with same base, add exponents"

Addition Rule (for logarithms)

$$\log_2 8 + \log_2 16 = \log_2 (8 \cdot 16)$$

$$\log_2 8 + \log_2 16 = \log_2 (128)$$

"when adding logs of the same base, multiply powers"

CK $3 + 4 = 7$ ✓

(Ex) $\log_3 14 + \log_3 2 = \log_3 28$

(Ex) $\log_5 18 = \log_5 3 + \log_5 6$

$18 = 3 \cdot 6$

(Ex) $\log 1000 = \log 10 + \log 100$

$3 = 1 + 2$ check ✓

Division Rule
(for Exponents)

$$\frac{3^4}{3^1} = 3^{4-1} = 3^3 = 27$$

"When divivinging powers
with same base,
subtract exponents"

Subtraction Rule
(for Exponents)

"When subtracting logs
of same base, <u>divide</u>
powers."

$$\log_2 16 - \log_2 8 = \log_2\left(\frac{16}{8}\right)$$

$$\log_2 16 - \log_2 8 = \log_2 (2)$$

$$4 - 3 = 1 \quad \text{Check} \checkmark$$

Ex $\log_3 19 - \log_3 4 = \log_3\left(\frac{19}{4}\right)$

Ex $\log 1000 - \log 100 = \log\left(\frac{1000}{100}\right)$

$$= \log(10) = 1$$

$$3 - 2$$

Ex $\log_5 X - \log_5 Y = \log_5\left(\frac{X}{Y}\right)$

Power to Power Rule
(for exponents)

"When raising powers to a power - multiply exponents"

$$(2^3)^4 = 2^{12}$$

Mult. Rule (for logs)
When multiplying a logarithm, raise the power "inside" the logarithm by this multiplier.

$$3\log_4 6 = \log_4 6^3$$

Ex $2\log_3 6 = \log_3 6^2 = \log_3 (36)$

Ex $\log_5 8^2 = 2\log_5 8$

Ex $\log_5 X^Y = Y\log_5 X$

Summary – 3 Log Rules

$\boxed{AR}$ $\log_b M + \log_b N = \log_b(MN)$

$\boxed{SR}$ $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$

$\boxed{MR}$ $N \log_b M = \log_b(M^N)$

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Basic Log Rules / Expanding Logarithmic Expressions *(page 1 of 5)*

Sections: Basic log rules, Expanding, Simplifying, Trick questions, Change-of-Base formula

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You have learned various rules for manipulating and simplifying expressions with exponents, such as the rule that says that $x^3 \times x^5$ equals x^8 because you can add the exponents. There are similar rules for logarithms.

Log Rules:

1) $log_b(mn) = log_b(m) + log_b(n)$

2) $log_b(^m/_n) = log_b(m) - log_b(n)$

3) $log_b(m^n) = n \cdot log_b(m)$

In less formal terms, the log rules might be expressed as:

1) Multiplication inside the log can be turned into addition outside the log, and vice versa.

2) Division inside the log can be turned into subtraction outside the log, and vice versa.

3) An exponent on everything inside a log can be moved out front as a multiplier, and vice versa.

Warning: Just as when you're dealing with exponents, the above rules work *only* if the bases are the same. For instance, the expression "$log_d(m) + log_b(n)$" cannot be simplified, because the bases (the "d" and the "b") are not the same, just as $x^2 \times y^3$ cannot be simplified (because the bases x and y are not the same).

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ACT If $\log_a X = s$ And $\log_a Y = t$ then $\log_a (XY)^2 = ?$

(A) $2(s+t)$	(B) $s+t$
(C) $4st$	(D) $2st$
(E) st	

ANS)

$$\log_a (XY)^2$$

$$2\left[\log_a (XY)\right]$$ MR for Logs

$$2\left[\underbrace{\log_a X}_{s} + \underbrace{\log_a Y}_{t}\right]$$ AR for logs

$\therefore$ $\boxed{\text{(A) } 2(s+t)}$