

BE - Geometry II Tuesday 4-19-11

① $f(x) = 5x + 10$

$f^{-1}(x) = ?$

② $g(x) = \frac{1}{5}x - 2$

$f[g(x)] = [f \circ g][x] = ?$

$f(x) = 5x + 10$

$f^{-1}(x) = \frac{1}{5}x - 2$

$g(x) = \frac{1}{5}x - 2$

$$\begin{aligned} f[g(x)] &= f\left(\frac{1}{5}x - 2\right) = 5\left(\frac{1}{5}x - 2\right) + 10 \\ &= 1x - 10 + 10 \end{aligned}$$

Look

$\boxed{= x}$

$$\begin{aligned} g[f(x)] &= g(5x + 10) = \frac{1}{5}(5x + 10) - 2 \\ &= x + 2 - 2 \end{aligned}$$

$\boxed{= x}$

* THE COMPOSITION OF INVERSE FUNCTIONS
ALWAYS = X

1.
Use the composition of two functions, both ways, as a test to see if they are inverses.

From last night's homework:

$$\textcircled{1} \quad f(x) = 2x - 4 \quad f^{-1}(x) = \frac{1}{2}x + 2$$

$$f[f^{-1}(x)] \Rightarrow f(x) = 2x - 4$$

$$f\left(\frac{1}{2}x + 2\right) = 2\left(\frac{1}{2}x + 2\right) - 4$$

$$= x + 4 - 4$$

$$= x \quad \checkmark$$

$$f^{-1}[f(x)] \Rightarrow f^{-1}(x) = \frac{1}{2}x + 2$$

$$f^{-1}(2x - 4) = \frac{1}{2}(2x - 4) + 2$$

$$= x - 2 + 2$$

$$= x \quad \checkmark$$

$y = 2x - 4$ and $y = \frac{1}{2}x + 2$ are inverses

Finding the Inverse of A Function

(1)

In[6]:= $f[x_] := 2x - 4$

In[7]:= $\text{InverseFunction}[f][x]$

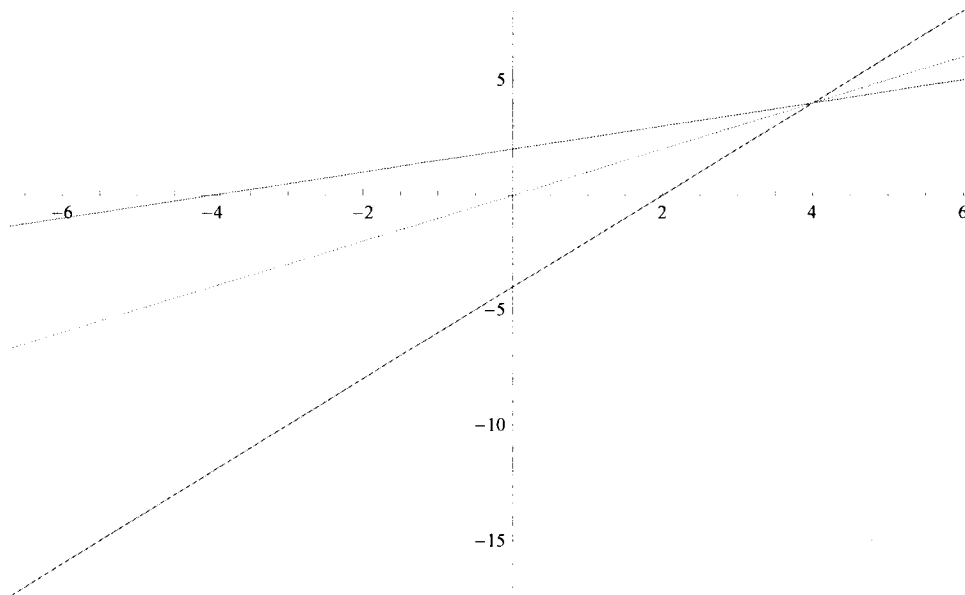
Out[7]:= $\frac{4 + x}{2}$

In[8]:=  Graph $y=2x-4$, $y=(4+x)/2$, $y=x$

↳ Plot

$\text{Plot}[\{-4 + 2 * x, (4 + x) / 2, x\}, \{x, -6.7, 6\}]$

Out[8]=



(2)

```
In[9]:= f[x_] :=  $\left(\frac{1}{2}\right) x + 3$ 
```

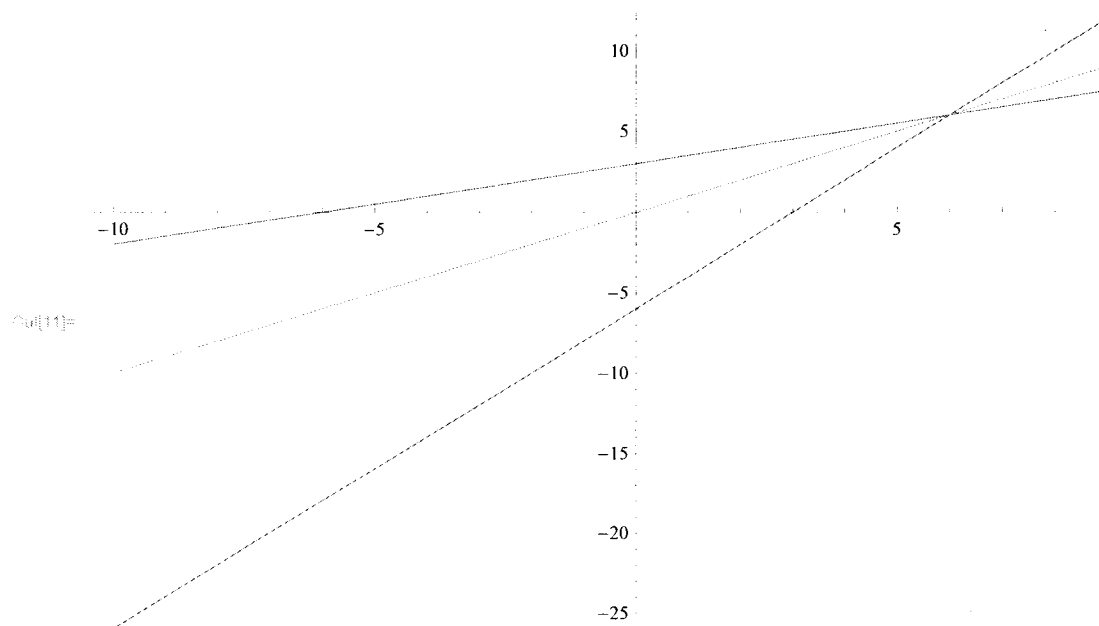
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In[10]:= InverseFunction[f][x]
```

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Out[10]= 2 (-3 + x)
```

```
In[11]:=  Graph  $y = (1/2)x + 3$ ,  $y = 2(-3 + x)$ ,  $y = x$ 
```

```
↳ Plot
```

```
Plot[{ $3 + x/2$ ,  $2 * (-3 + x)$ ,  $x$ }, {x, -10, 9}]
```



(3)

```
In[12]:= f[x_] := 4 x - 1
```

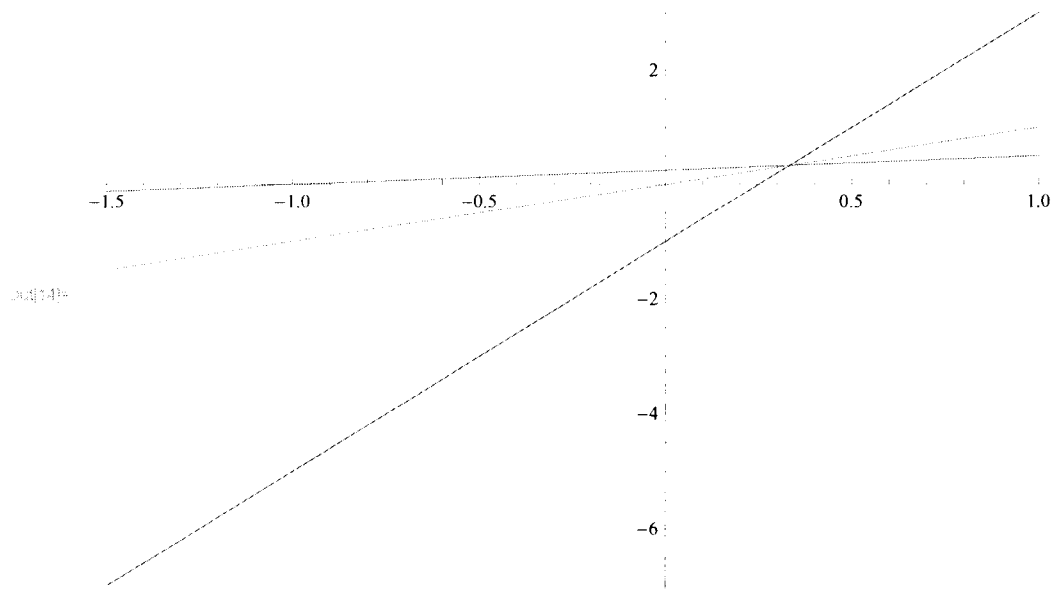
```
In[13]:= InverseFunction[f][x]
```

```
Out[13]=  $\frac{1+x}{4}$ 
```

```
In[14]:=  Graph y=4x-1, y=(1+x)/4, y=x
```

↳ Plot

```
Plot[{-1 + 4 * x, (1 + x) / 4, x}, {x, -1.5, 1}]
```



(4)

$$\text{In}[15]:= f[x_] := \left(\frac{-3}{4}\right)x + 6$$

$$\text{In}[18]:= \text{InverseFunction}[f][x]$$

$$\text{Out}[18]:= -\frac{4}{3}(-6 + x)$$

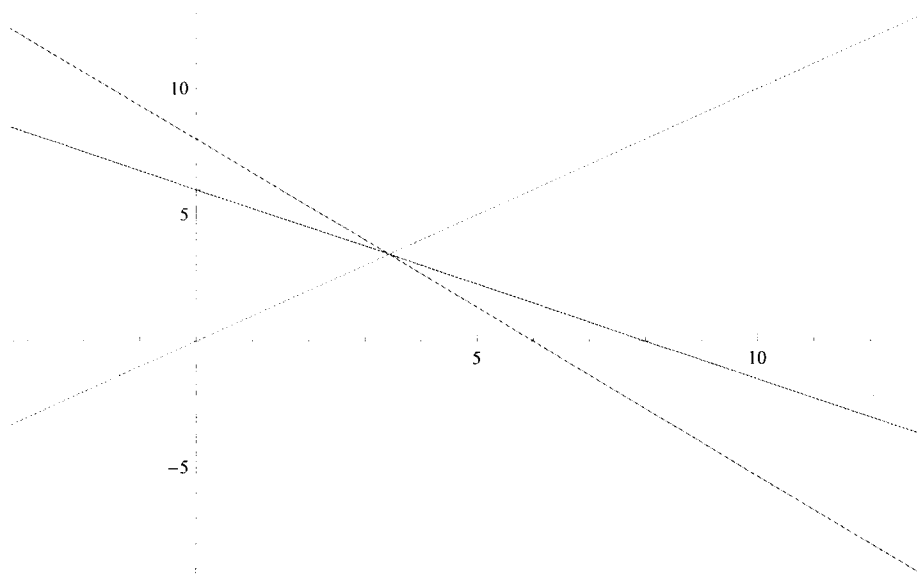
$$\text{In}[19]:= \text{Expand}\left[-\frac{4}{3}(-6 + x)\right]$$

$$\text{Out}[19]:= 8 - \frac{4x}{3}$$

$$\text{In}[20]:= \text{Graph } y = (-3/4)x + 6, \quad y = 8 - (4x)/3, \quad y = x$$

$$\rightarrow \text{Plot}$$

$$\text{Plot}[\{(6 - (3 * x) / 4, 8 - (4 * x) / 3, x), \{x, -3.3, 13\}]\}$$

$$\text{Out}[20]:=$$


ONE TO ONE
function

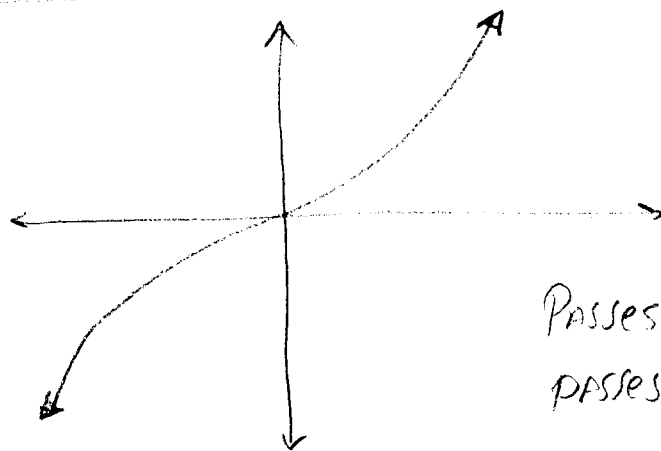
A function that has an
inverse that is also
a function.

Easy test:

① PASS VERTICAL LINE TEST
 $\Rightarrow$ FUNCTION (VLT)

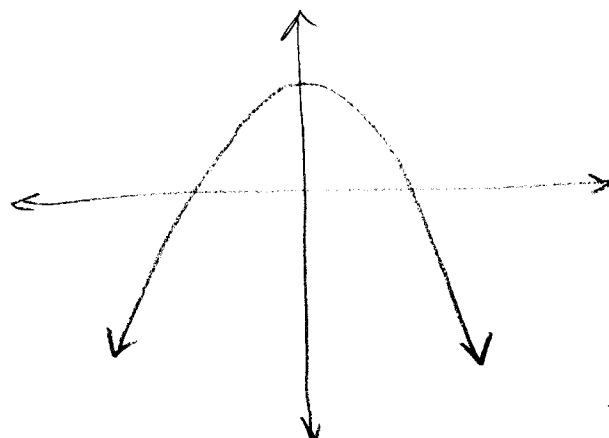
② PASS horizontal line test
 $\Rightarrow$ inverse is a function
(HLT)

(EX)



Passes VLT $\Rightarrow$ Function ✓
Passes HLT $\Rightarrow$ Inverse
is a
Function ✓

(EX)



Passes VLT
 $\Rightarrow$ Function ✓
FAILS HLT
 $\Rightarrow$ inverse is
NOT A
Function

Determine whether the pair of functions are inverses

$$f(x) = 6x + 2 \quad g(x) = x - \frac{1}{3}$$

$$\begin{aligned} f[g(x)] &\Rightarrow f\left(x - \frac{1}{3}\right) = 6\left(x - \frac{1}{3}\right) + 2 \\ &= 6x - 2 + 2 \\ &= 6x \quad \underline{\text{No}} \end{aligned}$$

Could stop there, it doesn't matter if $g[f(x)] = x$ or not.

Could check by finding $f^{-1}(x)$

$$y = f(x) = 6x + 2$$

→ swap ↙

$$x = 6y + 2$$

$$x - 2 = 6y$$

$$\frac{x-2}{6} = y \quad \neq x - \frac{1}{3} \quad \checkmark \text{ NOT INVERSES}$$

Converting From $^{\circ}\text{F}$ to $^{\circ}\text{C}$

$$y = f(x) = \frac{5}{9}(x - 32)$$

$x = \text{deg. Fahrenheit}$

$y = \text{deg. Celsius}$

OF WHAT use would be the inverse function?

$$f^{-1}(x) \Rightarrow x = \frac{5}{9}(y - 32)$$

$$\frac{9}{5}x = y - 32$$

$$\frac{9}{5}x + 32 = y$$

$\therefore$

$$y = f^{-1}(x) = \frac{9}{5}x + 32$$

$x = \text{deg. Celsius}$

$y = ^{\circ}\text{F}$

$$\text{CK: } x = 0^{\circ}$$

$$y = 32 \checkmark$$

$$x = 100$$

$$y = 180 + 32 = 212 \checkmark$$

Homework:

Use Algebra, determine whether each pair of functions are inverses

① $f(x) = x + 7$

$g(x) = x - 7$

② $g(x) = 3x - 2$

$f(x) = \frac{x - 2}{3}$

③ $f(x) = 3x + 4$

$g(x) = 3x - 4$

④ $g(x) = 2x + 8$

$f(x) = \frac{1}{2}x - 4$

Find the inverse of each function then graph the function, the inverse, and a dashed $y = x$ line of symmetry

⑤ $f(x) = x - 5$

⑥ $f(x) = \frac{4}{5}x - 7$