ACT MINI REVIEW: COMPLEX NUMBERS

$\sqrt{4} = 2$, $-\sqrt{4} = -2$, $\pm\sqrt{4} = \pm 2$

$\sqrt{-4} = ?$

There is no real number that can multiply by itself to get a negative since $-2 \cdot -2 = +4$ and $+2 \cdot +2 = +4$

Solution: define $i = \sqrt{-1}$

Then $\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{-1} \cdot \sqrt{4}$
$= i2$ or $\boxed{2i}$

Ex $\sqrt{-16} = \boxed{4i}$

$\sqrt{-25} = \boxed{5i}$

$\sqrt{-125} = \sqrt{-1}\sqrt{25}\sqrt{5}$
$= i \cdot 5 \cdot \sqrt{5}$
$= \boxed{5i\sqrt{5}}$

$\boxed{\text{Complex Numbers}}$

$$a + bi$$

REAL part   imaginary part

Ex $3 + 6i$

Ex $4 + 2i$

Ex $1 + 7i$

Ex $2 + 3i$

Ex $2 + 0i$

pure real

Ex $0 + 3i$

pure imaginary

For complex numbers to be equal, the real and imaginary parts must be equal.

Ex $3+2i = 3+2i$

Raising i to various powers

$i = \sqrt{-1} \therefore i^2 = (\sqrt{-1})^2 = -1$

$$\boxed{\begin{aligned} i^1 &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$$

$\div 4$, remainder 1
$\div 4$, remainder 2
$\div 4$, remainder 3
$\div 4$, remainder 0

Ex $i^5 = i$

$i^{12} = 1$

$i^{57} = i$

$i^{102} = -1$

Ex $(2 + 3i)(5 - 6i)$

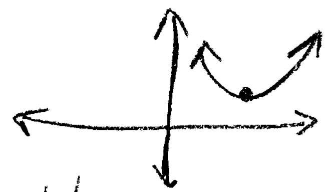
$10 - 12i + 15i - 18i^2$

$10 + 3i - 18(-1)$

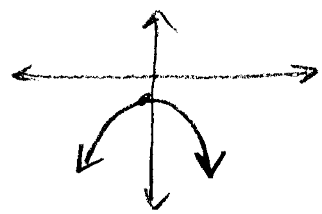
$\boxed{28 + 3i}$

$ax^2+bx+c=y$

A parabola that is a "smiley" above the X-axis



or a "frownie" below the X-axis



has a quadratic equation with no real solution.

$ax^2+bx+c=0$

The discriminant b^2-4ac is negative so the quadratic formula leads to a negative square root, that is, complex roots since

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Solve: $4x^2 = -3 - 4x$

$$4x^2 + 4x + 3 = 0$$

$a = 4 \qquad b^2 - 4ac$

$b = 4 \qquad (4)^2 - 4(4)(3)$

$c = 3 \qquad 16 - 48 = -32 = d$

$$x = \frac{-b \pm \sqrt{d}}{2a} = \frac{-4 \pm \sqrt{-32}}{8}$$

$\sqrt{32} = \sqrt{16}\sqrt{2}$

$$x = \frac{-4 \pm 4i\sqrt{2}}{8} = \boxed{\frac{-1 \pm i\sqrt{2}}{2} = x}$$

$$x = \left\{ -\frac{1}{2} + \frac{i\sqrt{2}}{2},\ -\frac{1}{2} - \frac{i\sqrt{2}}{2} \right\}$$

CK $\quad 4\left(-\frac{1}{2} + \frac{i\sqrt{2}}{2}\right)^2 + 4\left(-\frac{1}{2} + \frac{i\sqrt{2}}{2}\right) + 3 \stackrel{?}{=} 0$

$$4\left[\frac{1}{4} - \frac{i\sqrt{2}}{4} - \frac{i\sqrt{2}}{4} + \frac{i^2 \cdot 2}{4}\right] - 2 + 2i\sqrt{2} + 3 \stackrel{?}{=} 0$$

$$1 - 2i\sqrt{2} - 2 \quad - 2 + 2i\sqrt{2} + 3 \stackrel{?}{=} 0 \ \checkmark$$

ACT Practice

① For $i^2 = -1$, $(4+i)^2 = ?$

RACT

② For $i^2 = -1$,

$$\frac{1}{1+i} \cdot \frac{1-i}{1-i} = ?$$

③ $\log_x 36 = 2 \quad X = ?$

④ $(2^x)(4) = 8^3 \quad x = ?$

⑤ If $\log_a X = 5$ and $\log_a Y = t$, then $\log_a (XY)^2 = ?$

ANS

① $(4+i)(4+i) = 16 + 8i + i^2 = \boxed{15+8i}$

② $\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{1^2 - i^2} = \frac{1-i}{1-(-1)} = \boxed{\frac{1-i}{2}}$

③ $X^2 = 36 \therefore \boxed{X = 6}$

④ $2^x 2^2 = (2^3)^3 \therefore x + 2 = 9 \quad \boxed{X = 7}$

⑤ $\log_a (XY)^2 = 2\left[\log_a X + \log_a Y\right]$

$= \boxed{2[5+t]}$