BE-Alg. 2 | Monday 11-8-10

ACT
Barron's

(1.) $2i^4 - i^6 = ?$

(2.) If $p + 2\sqrt{x-1} = g$ and $g > p$, what is $x-1$ in terms of p and g?

(A) $\frac{\sqrt{g-p}}{2}$ (B) $\sqrt{\frac{g-p}{2}}$ (C) $\frac{g-p}{2}$

(D) $\frac{(g-p)^2}{2}$ (E) $\frac{(g-p)^2}{4}$

ANS

(1) $2i^4 - i^6$

$2(1) - (i^4)(i^2)$

$2 - (1)(-1) = 2 + 1 = \boxed{3}$

(2) $p + 2\sqrt{x-1} = g$

$-p \qquad\qquad -p$

$\frac{2\sqrt{x-1}}{2} = \frac{g-p}{2}$

$\sqrt{x-1} = \frac{g-p}{2} \quad \therefore \quad x-1 = \left(\frac{g-p}{2}\right)^2$

$\boxed{x-1 = \frac{(g-p)^2}{4}}$ ANS (E)

It is easy to use the ZPP (Zero Product Property) to solve a quadratic equation than can be factored. It requires you to put the QE (quadratic equation) in standard form $\Rightarrow ax^2+bx+c=0$

Perfect Square Trinomials (PST) are particularly easy to factor and solve:

"FLORIDA"

Since $(a+b)(a+b) = a^2+2ab+b^2$

$$x^2+10x+25=0$$

a, 2ab ✓, b

$$(x+5)^2=0$$

$$\boxed{x=\{5\}}$$

What if you had a PST but the right side was not zero?

EX 1
Pg 306

$$x^2+10x+25=49$$

$$(x+5)^2=49$$

Solve by using square roots

$$\sqrt{(x+5)^2}=\pm\sqrt{49}$$

Don't forget ±

$$x+5=\pm 7$$

$$x={}^-5\pm 7 \text{ or } \boxed{x=\{2,-12\}}$$

THE ability to take THE square root to solve Perfect Square Trinomials leads to the most powerful method of solving Quadratic Equations. A method that can solve ANY quadratic Equation, whether it can be factored or not!

THE idea is to use Algebra to force ANY quadratic to be A Perfect Square Trinomial. It is called "Completing THE Square"

THE leading coefficient must be ONE! If not, divide everything by a to make it ONE!!

EX

CTS

$$x^2 + 8x - 20 = 0$$

Not A PST, CTS. $a = 1$ ✓ MAKE A HOLE

$$x^2 + 8x + 16 = 20 + 16$$

Take $\frac{1}{2}$ of b and square it, ADD to both sides

$$\left(\tfrac{1}{2}\cdot 8\right)^2 = 4^2 = 16$$

$$(x+4)^2 = 36$$

$$x + 4 = \pm\sqrt{36}$$

$$x = -4 \pm 6$$

$$\boxed{x = \{2, -10\}}$$

CHECKS? ✓

Ex $x^2 + 3x - 18 = 0$

$$x^2 + 3x + \frac{9}{4} = 18 + \frac{9}{4} \qquad \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\left(\frac{1}{2}b\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{72}{4} + \frac{9}{4}$$

$$x + \frac{3}{2} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$$

$$x = -\frac{3}{2} \pm \frac{9}{2} \qquad \boxed{x = \{3, -6\}}$$

CHECKS? ✓

Ex $x^2 - 8x + 11 = 0$

$$x^2 - 8x + 16 = -11 + 16$$

$$(x-4)^2 = 5$$

$$x - 4 = \pm\sqrt{5}$$

$$\boxed{x = 4 \pm \sqrt{5}} \text{ or } \boxed{x = \{4+\sqrt{5}, 4-\sqrt{5}\}}$$

CK $4+\sqrt{5}$

$$(4+\sqrt{5})^2 - 8(4+\sqrt{5}) + 11 \stackrel{?}{=} 0$$

$$16 + 8\sqrt{5} + 5 - 32 - 8\sqrt{5} + 11 \stackrel{?}{=} 0$$

$$21 - 32 + 11 \stackrel{?}{=} 0 \checkmark$$

Check $4-\sqrt{5}$ too!

What if leading coefficient is NOT 1?

EX 5
Pg 309

Ch. 6-4 "Completing The Square"

$$\frac{2x^2}{2} - \frac{5x}{2} + \frac{3}{2} = \frac{0}{2} \qquad \div \text{ by } a$$

$$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16} \qquad \left(\frac{1}{2}\cdot\frac{5}{2}\right)^2 = \frac{25}{16}$$

$$\left(\frac{1}{2}b\right)^2$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{-24}{16} + \frac{25}{16} = \frac{1}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{1}{16}} = \pm\frac{1}{4}$$

$$x = \frac{5}{4} \pm \frac{1}{4} \qquad \boxed{x = \left\{\frac{3}{2}, 1\right\}}$$

CK
$x = \frac{3}{2}$

$$2\left(\frac{9}{4}\right) - 5\left(\frac{3}{2}\right) + 3 \stackrel{?}{=} 0$$

$$\frac{9}{2} - \frac{15}{2} + \frac{6}{2} \stackrel{?}{=} 0 \checkmark$$

CK
$x = 1$

$$2(1)^2 - 5(1) + 3 \stackrel{?}{=} 0 \checkmark$$

Ex 6
Pg 309

$$x^2 + 4x + 11 = 0$$

$$x^2 + 4x + 4 = -11 + 4$$

$$(x+2)^2 = -7$$

$$x+2 = \pm\sqrt{-7} = \pm(\sqrt{-1})\sqrt{7} = \pm i\sqrt{7}$$

$$\boxed{x = -2 \pm i\sqrt{7}} \quad \text{or} \quad \boxed{x = \{-2+i\sqrt{7}, -2-i\sqrt{7}\}}$$

SET NOTATION

CK $x = -2 + i\sqrt{7}$

$$(-2+i\sqrt{7})^2 + 4(-2+i\sqrt{7}) + 11 \stackrel{?}{=} 0$$

$$4 - \underline{4i\sqrt{7}} - 7 - 8 + \underline{4i\sqrt{7}} + 11 \stackrel{?}{=} 0$$

$$-15 + 15 \stackrel{?}{=} 0 \checkmark$$

Since:

$$(-2+i\sqrt{7})(-2+i\sqrt{7})$$

$$4 - 2i\sqrt{7} - 2i\sqrt{7} + (i)^2 7$$

$$4 - 4i\sqrt{7} - 7$$

CK $x = -2 - i\sqrt{7}$

$$(-2-i\sqrt{7})^2 + 4(-2-i\sqrt{7}) + 11 \stackrel{?}{=} 0$$

$$4 + \underline{4i\sqrt{7}} - 7 - 8 - \underline{4i\sqrt{7}} + 11 \stackrel{?}{=} 0$$

$$-15 + 15 \stackrel{?}{=} 0 \checkmark$$

Homework: Pg 311 # 32–36 Solve and Check