BE-PRECALC | MONDAY 11-23-09

① Solve $3^{x+2} = 3^{4x-1}$

Tip: if the bases are the same, how do the exponents compare if the left side = right side

---

ANS: if $3^{x+2} = 3^{4x-1}$

then $x+2 = 4x-1$

$3 = 3x$

$\boxed{1 = x}$

CK $3^{(1)+2} \stackrel{?}{=} 3^{4(1)-1}$ ✓

How would you solve $6^{3x} = 81$?

NOTE: bases are different!

We need a new tool...

If $y = f(x) = a^x$ An exponential function

The inverse function is found by swapping the x and y's $\Rightarrow$

$$f'(x) \Rightarrow x = a^y$$ y is the logarithm of x,

A logarithm is an exponent!!

A logarithm is an exponent!!

"Exponent Rules" will be useful.

Exponents must have a base. In logarithm problems, identify the exponents and bases.

↑

A logarithm is an exponent.

NOTATION: $\log_a x = y$ (Logarithmic Form)

The logarithm of base a of $x = y$
(exponent)
$= y$ or $a^y = x$ (Exponential Form)

(EX)

Logarithmic Form	Exponential Form
$\log_2 8 = 3$	$2^3 = 8$
$\log_4 16 = 2$	$4^2 = 16$
$\log_5 125 = 3$	$5^3 = 125$
$\log 100 = 2$	$10^2 = 100$

↑ "log" WITH NO INDEX IS UNDERSTOOD TO BE BASE 10

$\ln 7.389 = 2$	$e^2 = (2.71828)^2 \approx 7.389$

↑ "ln" MEANS NATURAL logarithm is base e

(EX)

Exponential	Logarithmic
$7^2 = 49$	$\log_7 49 = 2$
$9^3 = 729$	$\log_9 729 = 3$

Ch. 11-4 Logarithmic Functions

Ex 1→3
Pg 719

① Write in Exponential Form

$\Rightarrow \log_{125} 25 = \frac{2}{3} \Rightarrow 125^{\frac{2}{3}} = 25$

$\Rightarrow \log_8 2 = \frac{1}{3} \Rightarrow 8^{\frac{1}{3}} = 2$

---

② Write in logarithmic Form

$\Rightarrow 4^3 = 64 \Rightarrow \log_4 64 = 3$

$\Rightarrow 3^{-3} = \frac{1}{27} \Rightarrow \log_3 \left(\frac{1}{27}\right) = -3$

---

③ $\log_7 \frac{1}{49} = ?$

$\Rightarrow 7^x = \frac{1}{49}$ Since $49^{-1} = \frac{1}{49}$

$7^x = 49^{-1}$

$7^x = (7^2)^{-1}$

$7^x = 7^{-2}$

$\boxed{x = -2}$

Ck $\log_7 \frac{1}{49} = -2$

$7^{-2} = \frac{1}{49}$ ✓

Solving an Exponential Equation with logarithms:

(Ex) Intro. to 11-4 ⇒ Pg 718

Radioactive Half-life = time for $\frac{1}{2}$ of a radioactive material to decay

How long for 256,000 gms of Thorium-234 to decay to 1000 gms. Half-life of Th-234 = 25 days

---

$$N = N_0\left(\tfrac{1}{2}\right)^t$$

N = final amount
N_0 = initial amount
t = number of half-lives

$$1000 = 256{,}000\left(\tfrac{1}{2}\right)^t$$

$$\frac{1000}{256000} = \left(\tfrac{1}{2}\right)^t$$

$$\left(\frac{1}{256}\right) = \left(\frac{1}{2}\right)^t$$

$$\log_{\frac{1}{2}}\left(\tfrac{1}{256}\right) = t$$

$$\log_{\frac{1}{2}}\left(\left(\tfrac{1}{2}\right)^8\right) = t$$

$8 = t$ Definition of Logarithm

$8 \cdot 25 = 200$ DAYS

Since 256 → (2) 128 → (2) 64 → (2^6), $= 2^8$

The Properties of Logarithms (Exponents) Come Directly from Exponent Rule

MR If you are adding exponents, you must be multiplying bases!

DR If you are subtracting exponents, you must be dividing bases!

PPR If you are multiplying exponents, you must be raising a base to a power!

MR $\log_b MN = \log_b M + \log_b N$

DR $\log_b \frac{M}{N} = \log_b M - \log_b N$

PPR $\log_b M^P = P \log_b M$

} Pg 720

NOTE ⇒ if $\log_b M = \log_b N$ then $M = N$

SOLVE

Ex 5 Pg 721 (A) $\log_p 64^{\frac{1}{3}} = \frac{1}{2}$

$$p^{\frac{1}{2}} = 64^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

$$\left(p^{\frac{1}{2}}\right) = 4$$

$$\left(p^{\frac{1}{2}}\right)^2 = 4^2$$

$$\boxed{p^1 = 16}$$

(C) $\log_{11} x + \log_{11}(x+1) = \log_{11} 6$

ADDING EXPONENTS $\Rightarrow$ MR

$$\log_{11}(x)(x+1) = \log_{11} 6$$

$$\log_{11}(x^2+x) = \log_{11} 6$$

$$\therefore x^2 + x = 6$$

$$x^2 + x - 6 = 0$$

Sum = 1
Prod -6
$-2, +3$

$$(x-2)(x+3) = 0 \qquad \boxed{x = \{2, -3\}}$$

Homework: Pg 722-723 #7-15 odd