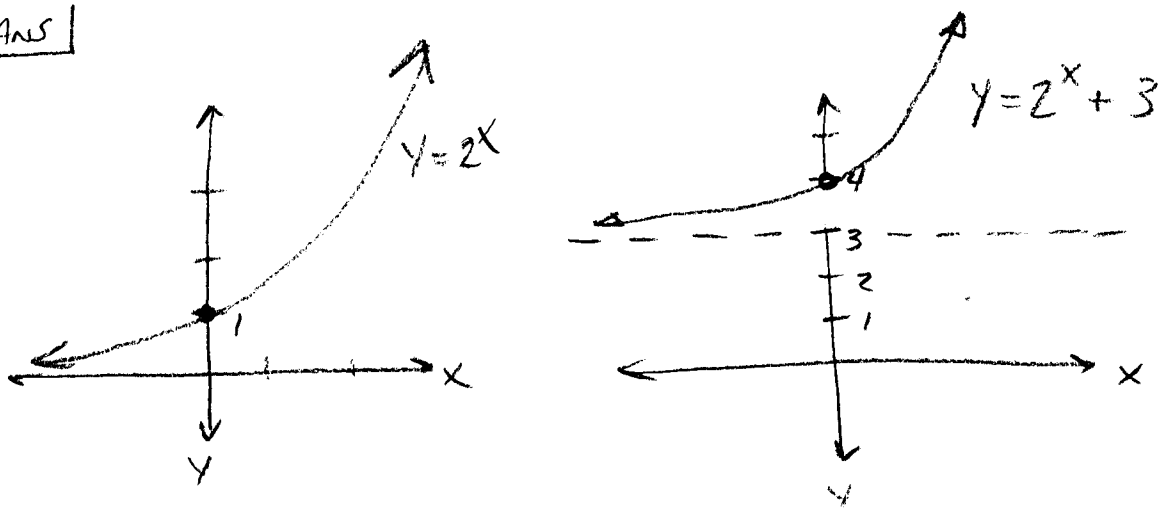
BE - Precalc / Wednesday 11-18-09

① Graph $y = 2^x$ and $y = 2^x + 3$ and compare them — how do their asymptotes differ?

Ans



- Homework review — p. 710 # 31

Does "compounding" interest have any limits? Can you compound hourly? Each second? Continuously?

Lets assume $r = 1$ and compare various compound periods. Lets also assume "P" = 1 so $A = P\left(1 + \frac{r}{N}\right)^{Nt}$

And $t = 1$ is $A = 1\left(1 + \frac{1}{N}\right)^{N \cdot 1}$

or $A = \left(1 + \frac{1}{N}\right)^{N}$

Try for various values of N

(A spreadsheet is good for this).

N	$\left(1 + \frac{1}{N}\right)^{N}$
1	2
10	⋮
100	⋮
⋮	⋮
⋮	?

There is a natural limit to the $(1+\frac{1}{N})^N$ as N gets bigger and bigger.

It is an irrational number named $e \approx 2.718281829...$

It is the base of natural logarithms $\Rightarrow$ LN on your calculator.

It is one of the most important and fundamental numbers in math (especially Calculus) and science.

The Exponential Growth and Decay Functions are often written in terms of e:

$$\boxed{A = A_0 e^{kt}}$$

$k = +$ for growth
$k = -$ for decay

AND $\boxed{A = Pe^{rt}}$ $\Rightarrow$ Continuous Compounding Interest formula

Numbers like e, π, NON-perfect square roots, SIN, COS, TAN, etc. ARE calculated using infinite series.

(Ex) $$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

! = factorial (Ex) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

The more terms you calculate, the more digits of e you get!

↑ EXCLAMATION point

Other examples:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots$$

$$\text{SIN}X = X - \frac{X^3}{3!} + \frac{X^5}{5!} - \frac{X^7}{7!} + \cdots$$

Ex 2
Pg 713

Compare $10,000 at 6.75% for 25 years when compounded semi-annually vs. continuous compounding

Semi-Annual	Continuous
$A = P\left(1+\frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$
$A = 10000\left(1+\frac{.0675}{2}\right)^{2 \cdot 25}$	$A = 10000(e)^{.0675 \cdot 25}$
$A = \$52,574.62$	$A = \$54,059.49$

Homework: Pg. 714 # 7, 8