BE - Precalc - Monday 11-16-09

Starting with 1 grain of rice on a standard 64 square chessboard, if you put 2 grains on the second square, 4 grains on the 3rd, etc. How many grains will you have after using all 64 squares?

Tip: $y = 2^x$ where y is total, x is number of squares

Origin of the problem:
http://en.wikipedia.org/wiki/Wheat_and_chessboard_problem

"While the story behind the problem changes from person to person, the fable usually follows the same idea:

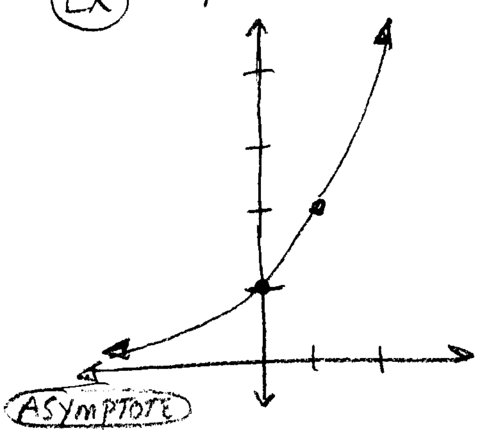
When the creator of the game of chess (in some tellings an ancient Indian mathematician, in others a legendary dravida vellalar named Sessa or Sissa) showed his invention to the ruler of the country, the ruler was so pleased that he gave the inventor the right to name his prize for the invention. The man, who was very wise, asked the king this: that for the first square of the chess board, he would receive one grain of wheat (in some tellings, rice), two for the second one, four on the third one, and so forth, doubling the amount each time. The ruler, who was not strong in mathematics, quickly accepted the inventor's offer, even getting offended by his notion that the inventor was asking for such a low price, and ordered the treasurer to count and hand over the wheat to the inventor. However, when the treasurer took more than a week to calculate the amount of wheat, the ruler asked him for a reason for his tardiness. The treasurer then gave him the result of the calculation, and explained that it would be impossible to give the inventor the reward. The ruler then, to get back at the inventor who tried to outsmart him, told the inventor that in order for him to receive his reward, he was to count every single grain that was given to him, in order to make sure that the ruler was not stealing from him."

Ans: $\sim 1.8 \times 10^{19}$ grains of rice. Example of exponential growth
Another modern example: Moore's Law

Ch. 11-2 Exponential Functions

If the variable of a function is an exponent, then the function is called an exponential function. The two basic types are exponential growth and exponential decay.

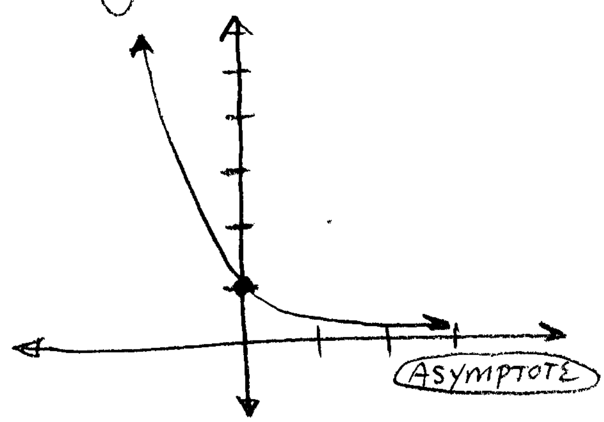
Ex $y = f(x) = 2^x$



Exponential growth

x	2^x
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

Ex $y = f(x) = \left(\frac{1}{2}\right)^x$



Exponential decay

x	$\left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

General Form of an Exponential Function For Growth or Decay of an Amount (A) for time (t) given a "starting value" of A_0 (when $t = 0$) is:

$$\boxed{A = A_0(1 \pm r)^t}$$

final amount (independent variable) "y"

(changes) number of time periods "x"

Amount of change (% as a decimal per time period

⊕ r ⇒ ⇒ growth

⊖ r ⇒ ⇒ decay.

growth

Ex $A_0 = 1$, * $r = 1$

$A = A_0(1+r)^t$

or $y = 1(1+1)^x$

$y = 2^x$

* 1 = 100% growth each "change" period

decay

Ex $A_0 = 1$, $r = \frac{1}{2}$

$A = A_0(1-r)^t$

or $y = 1(1-\frac{1}{2})^x$

$y = \frac{1}{2}^x$

$-\frac{1}{2}$ = 50% decay or loss each change period

The "base" of the exponential function must be >0, that is, it cannot be negative because the function would "flip" from $\oplus$ to $\ominus$ to $\oplus$ to $\ominus$ as x went from even to odd to even to odd

$$A = A_0\underbrace{(1 \pm r)}_{\text{BASE MUST BE POSITIVE}}{}^{t}$$

As t changes it may be even or odd

$$\boxed{\begin{array}{l} \text{BASE} > 1 \Rightarrow \text{growth} \\ 0 < \text{BASE} < 1 \Rightarrow \text{Decay} \end{array}}$$

$\text{BASE} = 1 \Rightarrow$ HORIZONTAL LINE (NOT A growth or Decay)

Simplifies to $y = b^x$

Ex (A) The student population at SMHS is increasing 3% per year. Model with an exponential function assuming the starting population is 600.

$$A = A_0(1+r)^t$$

A_0: starting population; r: growth; t: in years

$$r = 3\% = 03.0\% = .03$$

$$\therefore \boxed{A = 600(1.03)^t}$$

(b) Find the population in 15 years, 30 years.

$$A = 600(1.03)^{15}$$

$$A \cong 600(1.5580)$$

$$A \cong 934.8$$

$$\boxed{A \cong 935}$$

* +335

$$A = 600(1.03)^{30}$$

$$A = 600(2.4273)$$

$$A \cong 1456.4$$

$$\boxed{A \cong 1456}$$

* +521

Characteristic: *slow "wind-up" then very rapid change.

(EX)

(A) In business, depreciation is how fast an asset (something of value, like a truck), loses its value over time. Write an exponential function to model a depreciation of a truck at 15% per year ________ assuming its purchase price was $75,000.

Decay $\Rightarrow A = A_0(1-r)^t$

$15\% = .15$

$$\boxed{A = 75000(0.85)^t}$$

(B) What is the value of the truck after 7 years?

$$A = 75000(.85)^7$$

$$A = 75000(.3206)$$

$$\boxed{A = \$24{,}045}$$

- Homework: Pg 708 # 8, 9