BE-PRECALC | WEDNESDAY 10-28-09

ACT 2009-10 SAMPLE TEST

KMJ

① $|7-3|-|3-7|=?$

② $-3|-6+8|=?$

- Homework Review

ANS |

① $|4|-|-4|$

$4-4=\boxed{0}$

② $-3|2|$

$=-3(2)=\boxed{-6}$

Homework Review – Pg. 620 #12-17, 21

(12) $d = 13$, $Mid(1.5, 7)$ (13) $d = 2\sqrt{10}$, $Mid(0, 0)$

(14) $d = 8\sqrt{2}$, $Mid(4, 4)$ (15) $d = 3\sqrt{5}$, $Mid(2, -4.5)$

(16) $d = 4\sqrt{3}$, $Mid(5\sqrt{2}, -3)$ (17) $d = 16$, $(a, -1)$

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(21) $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $(a, -9), (-2a, 7)$

$d = 20 = \sqrt{(-7+9)^2 + (-2a-a)^2}$

$20 = \sqrt{16^2 + (-3a)^2}$

$(20)^2 = \left(\sqrt{16^2 + (-3a)^2}\right)^2$

$400 = 256 + 9a^2$

$144 = 9a^2$

$a^2 = \frac{144}{9}$ $\therefore a = \pm\frac{12}{3} \Rightarrow \boxed{a = \pm 4}$

Ch. 10-2 Circles.

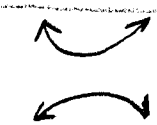
Geometry ⇒ A circle is one of the four "conic sections."

(cone) (cut-away)

They are: (P) Parabola 

(C) Circle 

(See pg. 623) (E) Ellipse 

(Flashlight Demo) (H) Hyperbola 

General Quadratic (2nd Degree) Function = Conic

Algebra ⇒ $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$B=0$ (P) A or $C=0$, ⇒ Parabola

$B=0$ (C) A and C = same sign, same value ⇒ CIRCLE

$B=0$ (E) A and C = same sign, different values ⇒ Ellipse

$B=0$ (H) A and C = opp. signs.

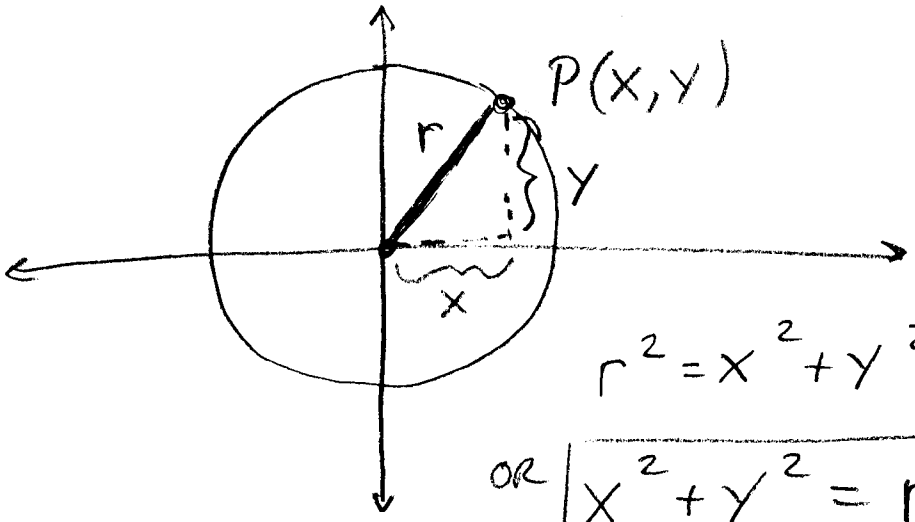
(H) ⟶ OR $A, C, D, E = 0$ $B \neq 0$

$\therefore Bxy + F = 0$

Ex) $y = \frac{1}{x}$ 

$A, B, C, = 0$ ⇒ 1st Degree ⇒ LINE

Lets try to discover the equation of a circle. Suppose you have a circle with its center at $(0,0)$ of radius r.



$$r^2 = x^2 + y^2$$

OR $\boxed{x^2 + y^2 = r^2}$

Equation of circle with center at $(0,0)$ and radius r

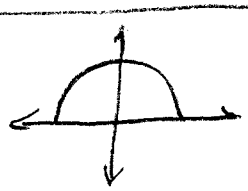
(Ex) $x^2 + y^2 = 16 \Rightarrow r = 4$

NOT A FUNCTION! Why not?

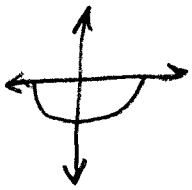
$$y^2 = 16 - x^2$$

$$\boxed{y = \pm\sqrt{16 - x^2}}$$

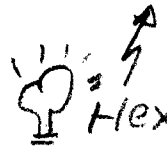
$\boxed{y = +\sqrt{16 - x^2}}$ FUNCTION

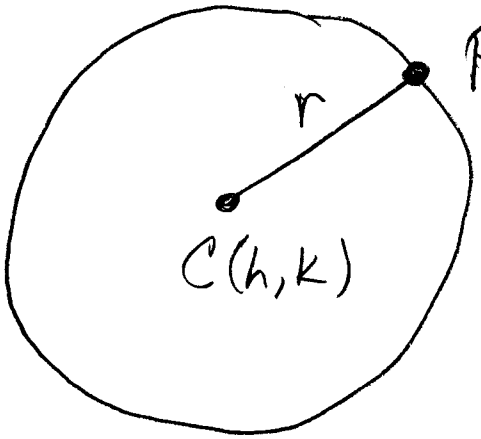


$\boxed{y = -\sqrt{16 - x^2}}$ FUNCTION



WHAT if the circle is NOT CENTERED ON the origin. SAY its center is AT (h, k)





Using the distance formula $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

or $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\Rightarrow \quad r^2 = (x-h)^2 + (y-k)^2$$

OR $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

General (h,k) form of the EQUATION of a circle with center AT (h,k) AND radius r

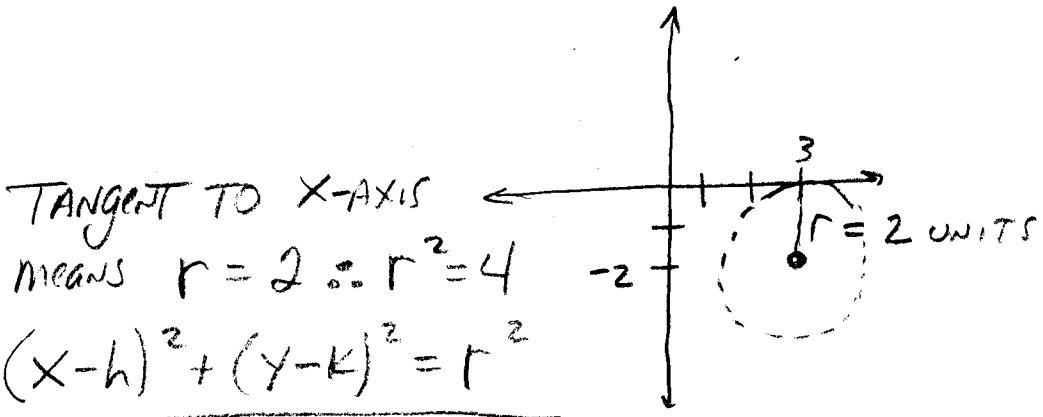
(EX) $(x-5)^2 + (y-2)^2 = 49$

CIRCLE WITH CENTER $(5,2)$, $r = 7$

(EX) $(x+3)^2 + (y-1)^2 = 15$

CIRCLE WITH CENTER AT $(-3,1)$, $r = \sqrt{15}$

Ex 1 Pg624 Write the standard form of an equation of a circle that is tangent to the x-axis and has its center at $(3, -2)$.



Tangent to x-axis means $r = 2 \therefore r^2 = 4$

$(x-h)^2 + (y-k)^2 = r^2$

$$\boxed{(x-3)^2 + (y+2)^2 = 4}$$

Convert standard form (h,k) $(x-h)^2+(y-k)^2=r^2$ to general form $Ax^2\ldots$ $x^2+y^2+Dx+Ey+F=0$

$A, C \Rightarrow$ same sign, same value, $\Rightarrow \div$ by A to make coefficient of x and $y = 1$

$B = 0$

$x^2 - hx - hx + h^2 + y^2 - ky - ky + k^2 = r^2$

$x^2 - 2hx + h^2 + y^2 - 2ky + k^2 - r^2 = 0$

$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2$

$$\boxed{x^2 + y^2 + Dx + Ey + F = 0}$$

General Form of a Circle

Making a Perfect Square Trinomial ⇒ Completing the Square

$a^2 \pm 2ab + b^2$

$\downarrow \qquad \swarrow$

$(a \pm b)^2$

(Ex) $x^2 + 6x + 9$

$\searrow \; 2 \cdot x \cdot 3 \; \swarrow$

$(x+3)^2 \Rightarrow (x+3)(x+3)$

- Make sure a is 1, if not $\div$ by a
- Take $\frac{1}{2}$ of b and square it, follow GRE
- Take $\sqrt{} \Rightarrow (a \pm b)^2 \Rightarrow$ factors

(Ex) $x^2 - 2x - 9 = 0$

$x^2 - 2x + \boxed{1} = 9 + \boxed{1}$ $\quad \frac{1}{2} \cdot 2 = 1, \; 1^2 = 1$

$\downarrow \qquad \swarrow$

$(x-1)^2 = 10$

$x - 1 = \sqrt{10}$

$\boxed{x = 1 \pm \sqrt{10}}$

Completing the square is useful for converting from the General Form of a Circle to the Standard Form

$x^2 + y^2 + Dx + Ey + F = 0$

to $\quad (x-h)^2 + (y-k)^2 = r^2$

Ex 3 Pg 626

Is $2x^2 + 2y^2 - 4x + 12y - 18 = 0$ THE EQUATION OF A CIRCLE?

$Ax^2 + \cancel{Bxy}^{0} + Cy^2 + Dx + Ey + F = 0$ ✓

A AND C ARE SAME SIGN, SAME VALUE = CIRCLE

(A) Change to (h,k) Form (STANDARD) (B) FIND (h,k) AND $C(h,k)$ ↗ COORDINATES OF CENTER

(C) GRAPH

Get $A = 1 \Rightarrow \div$ by 2

$$x^2 + y^2 - 2x + 6y - 9 = 0$$

$$(x^2 - 2x + \boxed{1}) + (y^2 + 6y + \boxed{9}) = 9 + \boxed{1} + \boxed{9}$$

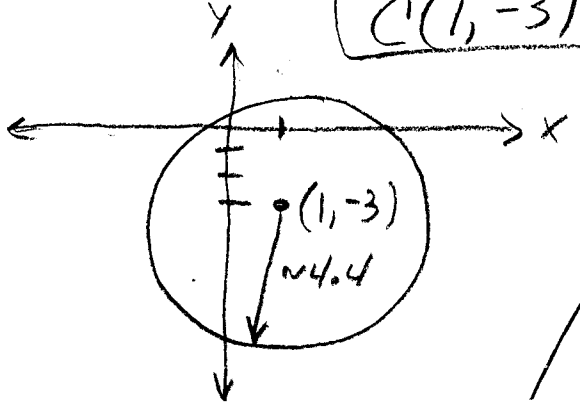
$\left[\frac{1}{2}(-2)\right]^2$ ↗ $\left[\frac{1}{2}(6)\right]^2$ ↗

$$(x-1)^2 + (y+3)^2 = 19$$

$h = 1$ $\quad k = -3$ $\quad r = \sqrt{19}$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$ STANDARD Form

$\boxed{C(1,-3) \quad r = \sqrt{19}}$ $\quad \sqrt{19} \cong 4.4$



Homework:
Pg. 627-628
15, 17, 19, 23