BE-Alg. 2 & Precalc | Monday 9-21-09

"Real" ACT 2nd Ed. PT3

1. On the real number line, what is the midpoint of -5 and 17?

2. If $3\frac{3}{5} = x + 2\frac{2}{3}$, then $x = ?$

3. A system of linear equations is shown below.

$$3y = -2x + 8$$
$$3y = 2x + 8$$

Which of the following describes the graph of this system of linear equations in the standard (x, y) coordinate plane?

(A) Two distinct intersecting lines.
(B) Two parallel lines with positive slope.
(C) Two parallel lines with negative slope.
(D) A single line with positive slope.
(E) A single line with negative slope.

- Time permitting ⇒ review HW 5 & Quiz 4
- Best practice problems ⇒ Real ACT
- Best topic-by-topic study guide ⇒ Barrons

Ch 4-1 Polynomial Functions

Mini Review ⇒ $\sqrt{16} = 4 = \frac{4}{1}$ RATIONAL

$\sqrt{15} \cong 3.87... =$ IRRATIONAL

$\{$RATIONAL, IRRATIONAL$\}$ = Real Numbers

↑ term. or Rep. decimal ↑ ∞ Non-Rep. Decimal

$\sqrt{16} = 4$ Principal square root

$-\sqrt{16} = -4$

$\pm\sqrt{16} = \pm 4$

$\sqrt{-16} =$ CANNOT BE FOUND USING Reals since $-4 \cdot -4 = 16$, $4 \cdot 4 = 16$

But $\sqrt{-16} = \sqrt{16}\sqrt{-1}$

Define $i = \sqrt{-1} \Rightarrow \sqrt{-16} = 4i$ OR $0 + 4i$

Real Part Imaginary Part

$\{$Reals, imaginary$\}$ = Complex Numbers

$i^1 = \sqrt{-1} = \boxed{i}$

$i^2 = \sqrt{-1} \cdot \sqrt{-1} = (\sqrt{-1})^2 = \boxed{-1}$ By Definition of $\sqrt{\ }$

$i^3 = i^2 i = -1 \cdot \sqrt{-1} = -\sqrt{-1}$ OR $\boxed{-i}$

$i^4 = i^3 \cdot i = -i \cdot i = -i^2 = -(\sqrt{-1})^2 = \boxed{1}$

or $i^2 \cdot i^2 = -1 \cdot -1 = 1$

MONOMIAL — A number, variable, or product of A number AND one or more variables.

(Ex) $x, 4, 3x, 5x^2y$

(NOT EX) $\frac{1}{x}, \frac{3}{x+4}, \frac{5x}{2y}$

POLYNOMIAL — ONE or more MONOMIALS separated by $\pm$

(Ex) $3, x, x+3, 2x^2+4x-2$

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degree of a monomial — SUM OF THE EXPONENTS of THE variables

(Ex) degree 1	degree 2	degree 3
$2x, 5y$	$3x^2, 2xy$	$5x^3, 7x^2y, 2xyz$

NOTE: Numbers HAVE Degree = 0

Degree of A POLYNOMIAL — the same as the monomial term of the highest degree

(Ex) degree 2 $\Rightarrow 3x^2+2x+4$

POLYNOMIALS ARE WRITTEN: highest degree $\rightarrow$ lowest (numbers always last)

Leading Coefficient — the coefficient of the highest degree term. (Ex) $7x^3+2x^2-3x+4$

LEADING coefficient = +7

FTOA Fundamental Theorem of Algebra

Every polynomial equation of

$$\left(a_0 x^N + a_1 x^{N-1} + \dots a_{N-1} x^1 + a_N = 0\right)$$

degree N has N complex roots.

maximum number of times the graph of the polynomial crosses the x-axis (each crossing is a real root)

See page 207

"Even" functions or

"Odd" functions or Joan Travolta functions
(degree 1, 3, 5, ...)

must cross at least once $\Rightarrow \geq 1$ real root

If $a, b, c, \dots$ are roots of the polynomial,

then you can write $(x-a)(x-b)(x-c)\dots = 0$

Ex 3 Pg 208 WRITE A POLYNOMIAL EQUATION WITH roots $2, 4i, -4i$

$$\Rightarrow (x-2)(x-4i)(x+4i)=0$$

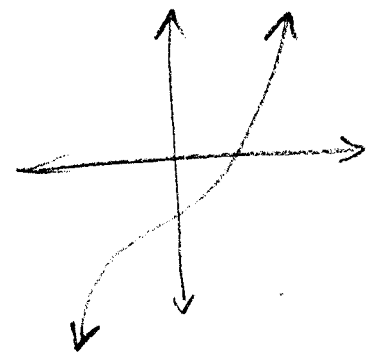
WATCH OUT $i^2=-1$

$$x-2\left[x^2+4ix-4ix-16i^2\right]=0$$

$$x-2\left[x^2+16\right] = x^3-16x-2x^2-32=0$$

$$\therefore \boxed{x^3-2x^2-16x-32=0}$$

"ODD" FUNCTION WITH 1 real root



Ex 4 pg208 How many complex roots does $9x^4-35x^2-4=0$ have?

degree $=4$ $\therefore$ $\boxed{4 \text{ complex roots}}$

$\boxed{\text{QUADRATIC EQUATIONS}}$ Ch. 4-2

Standard QE $\Rightarrow ax^2 + bx + c = 0$

QF $\Rightarrow ax^2 + bx + c = y$ or

PARABOLA

METHODS to SOLVE: factoring $\Rightarrow$ if factorable

completing the square $\Rightarrow$ always

If the roots are real (crosses X-AXIS twice) and are RATIONAL, $ax^2+bx+c=0$ can be solved by factoring $\Rightarrow$ "magic number" METHOD

* PUT IN STANDARD FORM FIRST!

"Magic Number" method for factoring quadratics:

(EX) Solve by factoring: $2x^2+11x=21$

$\Rightarrow\ 2x^2+11x-21=0$

Standard form
$a=2,\ b=11,\ c=-21$

sum $\Rightarrow b=11$
prod $\Rightarrow ac=-42$

$-3\quad +14$

Find the 2 factors, the magic numbers that add to $+11$ and multiply to -42

$2x^2-3x+14x-21=0$

Rewrite middle term $+11x$

$-3x+14x$

magic numbers

$(2x^2-3x)+(14x-21)=0$ Factor by grouping

$x(2x-3)+7(2x-3)=0$ Factor common group $(2x-3)$

$(2x-3)(x+7)=0$

Use Zero Product product to find the two roots

$\boxed{x=\left\{-7,\ \frac{3}{2}\right\}}$

$2x-3=0 \qquad x+7=0$
$2x=3 \qquad x=-7$
$x=\frac{3}{2}$

Check solution by substituting into original equation

Homework: Solve by factoring Pg 219 # 5, 9, 12, 13, 14