BE-Alg 2 | Monday 11-23-09

① Define a function.

② Is $\{(-2,-1),(-1,1),(0,3),(1,5),(2,7)\}$ a function?

③ Graph: $y = x$

The inverse of a function is found by switching every x and y. It has its own notation, if $y = f(x)$ the inverse of $f(x)$ is $y = f^{-1}(x)$

The inverse of $\{(-2,-1),(-1,1),(0,3),(1,5),(2,7)\}$ is $\{(-1,-2),(1,-1),(3,0),(5,1),(7,2)\}$

Suppose, instead of 5 ordered pairs, I told you $y = f(x) = 2x + 3$
How do you find $y = f^{-1}(x)$?

Take $\boxed{y = f(x) = 2x + 3}$, swap $x \to y$

$f^{-1}(x) \Rightarrow x = 2y + 3$, now solve for y

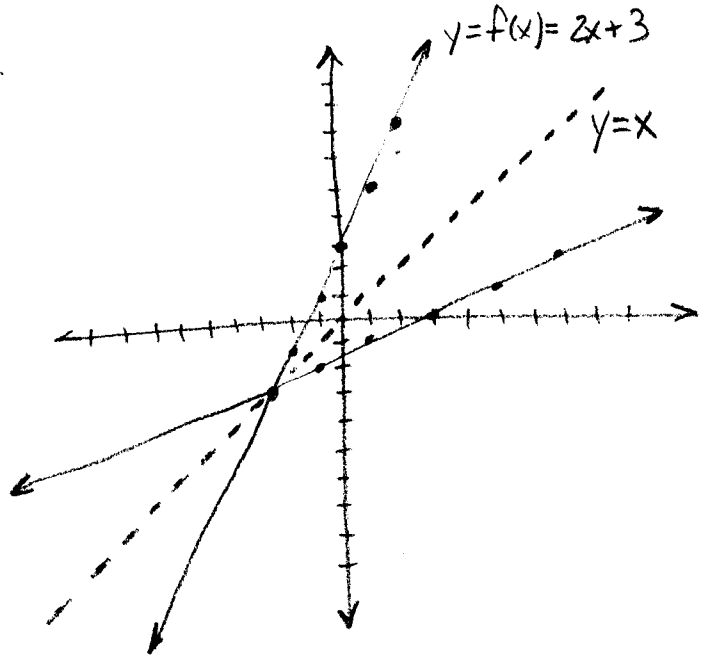
$$x - 3 = 2y$$

$$\frac{x-3}{2} = y$$

$$\boxed{y = f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}}$$

x	$f(x)$
-2	-1
-1	1
0	3
1	5
2	7

x	$f^{-1}(x)$
-1	-2
1	-1
3	0
5	1
7	2



INPUT →	OUTPUT →	INPUT →	OUTPUT
-3	-3	-3	-3

$y = x$

$y = 1x + 0$ AXIS OF SYMMETRY

The inverse of a relation may or may not be a function. For a function to have an inverse function, they must be symmetrical across the line $y = x$.

Algebraically $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

Ex $f(x) = 2x + 3$ $\quad f'(x) = \frac{1}{2}x - \frac{3}{2} = g(x)$

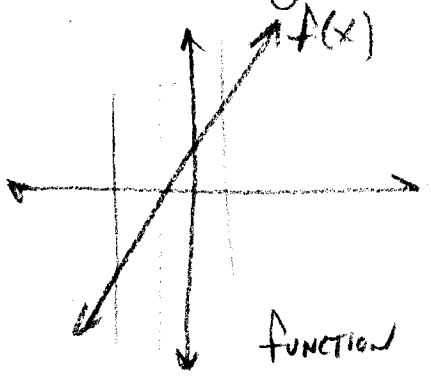
$(f \circ g)(x) = 2\left(\frac{1}{2}x - \frac{3}{2}\right) + 3$

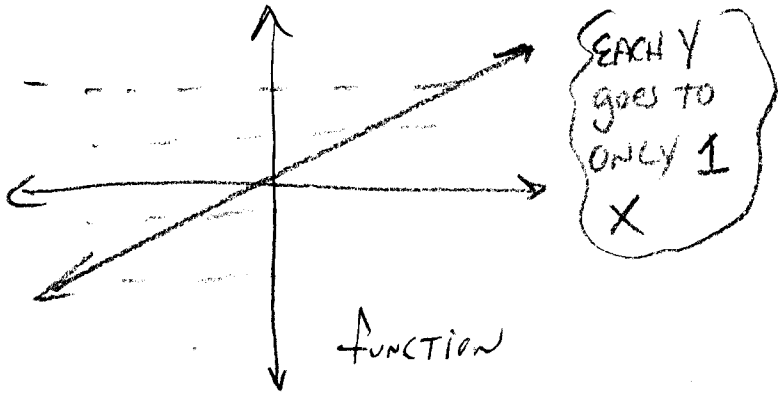
$= x - 3 + 3$ ✓

$(g \circ f)(x) \longrightarrow \frac{1}{2}(2x+3) - \frac{3}{2}$

$x + \frac{3}{2} - \frac{3}{2} = x$ ✓

Graphically ⇒ Pass the Horizontal Line Test





Ch 7-8 Inverse Functions and Relations

Ex 2
Pg 391 Find the inverse of $y = f(x) = \frac{x+6}{2}$

$$f^{-1}(x) \Rightarrow x = \frac{y+6}{2} \quad \text{Solve for } y$$

$$2x = y + 6$$

$$\therefore \boxed{y = f^{-1}(x) = 2x - 6} \quad \boxed{y = f(x) = \frac{x+6}{2}}$$

Ck: $f[f^{-1}(x)]$ or $(f \circ f^{-1})(x)$

$$f(x) = \frac{x+6}{2}$$

$$f(2x-6) = \frac{(2x-6)+6}{2} = x \checkmark$$

$f^{-1}[f(x)]$ or $(f^{-1} \circ f)(x)$

$$f^{-1}(x) = 2x - 6$$

$$f^{-1}\left(\frac{x+6}{2}\right) = 2\left(\frac{x+6}{2}\right) - 6$$

$$= x \checkmark$$

See Pg 391 for graphs
$\Rightarrow$ symmetrical across the line $y = x$ ✓

Summary

If $f(x)$ and $f^{-1}(x)$ are inverse functions:

$\Rightarrow$ then $f(a) = b$ and $f^{-1}(b) = a$

$\Rightarrow$ the graphs of $f(x)$ and $f^{-1}(x)$ are symmetric across the line $y = x$

$\Rightarrow$ $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

$\Rightarrow$ Pass Vertical and Horizontal Line Tests

* Homework: - Pg 393 # 8 to 11

The $( )^{-1}$ notation is used to represent many inverses, look at your scientific calculator:

x^{-1}, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

To find the inverse of $f(x)$, rewrite and swap x and y then solve for y.