$\boxed{\text{BE - Alg. 2}}$ MONDAY 11-2-09

ACT 2009 & 2010 PRACTICE TEST

(1) WHAT RATIONAL NUMBER IS halfway between $\frac{1}{5}$ AND $\frac{1}{3}$?

(2) If $a < b$, then $|a-b|$ IS EQUIVALENT TO which of the following?

(A) $a+b$ (B) $-(a+b)$ (C) $\sqrt{a-b}$ (D) $a-b$ (E) $-(a-b)$

ANS (1) $\frac{1}{5} + \frac{1}{3}$

$\frac{3}{15} + \frac{5}{15} = \frac{8}{15} \cdot \frac{1}{2} \therefore \boxed{\frac{4}{15}}$ is $\frac{1}{2}$ WAY

(2) If $a < b \Rightarrow$ TAKING b from a will be negative, but the absolute value will turn this difference positive.

$-(a-b)$ will have the same result, changing the sign of the difference.

$= |a-b|$ if $a < b$ $\therefore$ ANSWER (E)

Reviews of how to solve QUADRATIC EQUATIONS (2ND degree)

if they can be FACTORED ⟹ Roots ALWAYS RATIONAL

(Clean-up) • Combine like terms, put in STANDARD form, $(ax^2+bx+c=0)$ and factor a GCF if possible.

(FACTOR) • FACTOR using the magic number method.

(SOLVE) • Write solutions using ZPP. (Zero product property)

(Check) • Check solutions

(Ex) SOLVE

$$3x^2+5x+10=-16x-20 \quad \Big\}\text{ COMBINE LIKE TERMS}$$

$$3x^2+21x+30=0 \quad \Big\}\text{ STANDARD Form}$$

$$3(x^2+7x+10)=0 \quad \Big\}\text{ GCF}$$

$$ax^2+bx+c \quad \Big\}\text{ FACTOR, MAGIC NUMBER METHOD}$$

sum = b = 7

prod = ac = 10

+2 +5

$$\underline{\underline{3}}(x+2)(x+5)=0$$

Since $a=+1$ Check w/ FOIL

NOTE: WHILE NECESSARY, this 3 does NOT AFFECT SOLUTION

$$\boxed{x=\{-2,-5\}}$$

ZPP } SOLVE

} CHECK

CK

$x=-2$: $3(-2)^2+5(-2)+10 \stackrel{?}{=} -16(-2)-20$

$12-10+10 \stackrel{?}{=} 32-20$ ✓

$x=-5$: $3(-5)^2+5(-5)+10 \stackrel{?}{=} -16(-5)-20$

$75-25+10 \stackrel{?}{=} 80-20$ ✓

Chapter 6-3 Solving Quadratic Equations By Factoring

Be alert to the special form of a Perfect Square Trinomial ⇒ you can factor these using mental math.

Ex 2 Pg 302 Solve $x^2 - 16x + 64 = 0$

$2 \cdot x \cdot 8$ ✓

$(x-8)^2 = 0$

or $(x-8)(x-8) = 0$ $\boxed{x = \{8\}}$ Did you check this?

This type of quadratic, a Perfect Square, is said to have a Double Root.

Note: $\sqrt{x^2 - 16 + 64} = (x-8)$

Because $(x-8)(x-8) = x^2 - 16x + 64$

Suppose the "c" term in $ax^2 + bx + c = 0$ is 0. Then the "GCF" will be all you need to factor to solve the quadratic.

Ex 1 Pg 301 $x^2 = 6x$

$x^2 - 6x = 0$

$x(x-6) = 0$

or $(x-0)(x-6) = 0$ $\boxed{x = \{0, 6\}}$ Checks?

Standard Form

$x^2 - 6x + 0 = 0$

ZPP

What if "b" = 0 in $ax^2+bx+c=0$?

Easiest way to solve is "Isolate" the x^2 term and take the square root of both sides.

Ex

$$4x^2 = 30$$

$$4x^2 - 30 = 0$$

Standard Form $4x^2+0x-30=0$
$a=4$ $b=0$
$c=-30$

$$\quad +30 \quad +30$$

$$\frac{4x^2}{4} = \frac{30}{4}$$

$$x^2 = \frac{15}{2}$$

$$\sqrt{x^2} = \pm\sqrt{\frac{15}{2}}$$

$$\boxed{x = \pm\sqrt{\frac{15}{2}} = \pm\frac{\sqrt{30}}{2}}$$

Most Common Step is $x^2=\frac{15}{2}$ To $x=\pm\sqrt{\frac{15}{2}}$

Since $\sqrt{x^2} = |x|$

$|x| = \sqrt{\frac{15}{2}}$

$+(x) = \frac{15}{2}$

$-(x) = \frac{15}{2}$

$x = -\frac{15}{2}$

Simplify – RAT. THE BOT.

$$\frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{30}}{2}$$

CK $4\left(\sqrt{\frac{15}{2}}\right)^2 \stackrel{?}{=} 30$

$4\cdot\left(\frac{15}{2}\right) \stackrel{?}{=} 30$ ✓

$4\left(-\sqrt{\frac{15}{2}}\right)^2 \stackrel{?}{=} 30$

$4\left(-1\cdot\sqrt{\frac{15}{2}}\right)^2 \stackrel{?}{=} 30$

$4\left(\frac{15}{2}\right) \stackrel{?}{=} 30$ ✓

If you solve a quadratic by taking the square root of both sides, it may have complex roots.

Factorable quadratics will never have complex roots, only real roots. Many, many quadratics are not factorable — we will get to those soon.

Ex $\quad 5x^2 + 20 = 0 \quad \Rightarrow b = 0$

$$\frac{5x^2}{5} = \frac{-20}{5}$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4} = \pm\sqrt{-1}\sqrt{4}$$

$$\boxed{x = \pm 2i}$$

CK

$$5(2i)^2 + 20 \stackrel{?}{=} 0 \qquad 5(-2i)^2 + 20 \stackrel{?}{=} 0$$

$$5(4)(i^2) + 20 \stackrel{?}{=} 0 \qquad 5(4)(i^2) + 20 \stackrel{?}{=} 0$$

$$20(-1) + 20 \stackrel{?}{=} 0 \checkmark \qquad 20(-1) + 20 \stackrel{?}{=} 0 \checkmark$$

Summary: Solving $ax^2+bx+c=0$ by factoring and solving the two special cases $b=0$ and $c=0$.

NOTE: $a=0 \Rightarrow$ Linear Equation

(1.) Factorable || Roots always rational

- Cleanup $\Rightarrow$ Combine Like Terms, Standard Form, GCF
- Factor $\Rightarrow$ Magic Numbers Unless • Perfect Square Trinomial $\Rightarrow$ Double Root OR • $c=0$ $\Rightarrow$ GCF only
- Solve $\Rightarrow$ ZPP
- Checks

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(2) May have complex roots

If $b=0$, isolate x^2 & take $\sqrt{}$ of both sides, don't forget $x=\pm\sqrt{}$

Practice using ZPP and mental math to name the roots (solutions):

Ex $(x+5)(x-3)=0$ $\boxed{x=\{-5,3\}}$

Ex $(2x-3)(5x+6)=0$ $\boxed{x=\left\{\frac{3}{2},\frac{-6}{5}\right\}}$

Why?

$$\begin{aligned} 2x-3&=0 \\ +3\ \ &\ \ +3 \\ \frac{2x}{2}&=\frac{3}{2} \end{aligned} \qquad \begin{aligned} 5x+6&=0 \\ -6\ \ &\ \ -6 \\ \frac{5x}{5}&=\frac{-6}{5} \end{aligned}$$

Name the solutions:

$(x-1)(6x+7)=0$ $\boxed{x=\{1, \frac{-7}{6}\}}$

$(3x+4)(2x-1)=0$ $\boxed{x=\{\frac{-4}{3}, \frac{1}{2}\}}$

order?

Can you go backwards? If you know the solutions can you write the original quadratic?

Ex $\boxed{x=\{2,3\}} \Rightarrow (x-2)(x-3)=0$

$\boxed{x^2-5x+6=0}$

Ex $\boxed{x=\{-5,1\}} \Rightarrow (x-(-5))(x-1)=0$

$(x+5)(x-1)=0$

$\boxed{x^2+4x-5=0}$

Ex $\boxed{x=\{\frac{-4}{3}, \frac{1}{2}\}}$ $(x-(-\frac{4}{3}))(x-\frac{1}{2})=0$

$(x+\frac{4}{3})(x-\frac{1}{2})=0$

Look

$3(x+\frac{4}{3})=0\cdot 3$

$3x+4=0$

$(3x+4)(2x-1)=0$?

$\boxed{6x^2+5x-4=0}$ cks?

Homework: Pg. 303 # 4–12.
(Ch. 6-3)