BE-1B TUESDAY 4-8-08

IN 2000, the US POPULATION WAS $\approx 2.8 \times 10^8$.
The GROWTH RATE WAS $\approx 0.85\%$.

① WRITE AN EQUATION to REPRESENT the US POPULATION SINCE the year 2000.

② USE the EQUATION TO FIND THE POPULATION in the year 2010

• TAKE OUT homework

SOLUTIONS:

① $y = C(1+r)^t$

$\therefore \boxed{y = 2.8 \times 10^8 (1 + 0.0085)^t}$

② $y = 2.8 \times 10^8 (1.0085)^{10}$

$y = 2.8 \times 10^8 (1.0883)$

$\boxed{y = 3.05 \times 10^8}$

A special case of exponential growth is "compound interest." The following terms are used: $y = C(1+r)^t$

$$A = P(1+r)^t$$

A = the total Amount of the investment

P = Principal, the starting Amount of the investment

r = the yearly or annual interest rate.

The term compound interest means the bank puts the interest into the account before the year is up, so this new Amount earns interest longer. But the formula has to be adjusted, to do this the rate is divided by N, the number of times (at regular intervals) the interest is put into the account and the time t is multiplied by N so it has the total number of time periods.

$$A = P\left(1+\frac{r}{N}\right)^{Nt}$$

Compound Interest Formula

NOTE: IF INTEREST IS "COMPOUNDED ANNUALLY", THEN

$$A = P(1+r)^t$$

EX) $1000 is invested.
Interest rate is 7% compounded monthly.
Find value of investment after 5 years.

$P = \$1000$ $\quad r = 7\%$ per year $= .07$

↑ understood to be per year

$N = 12$ since there are 12 months/year

$$A = P\left(1 + \frac{r}{N}\right)^{Nt}$$

$$A = 1000\left(1 + \frac{.07}{12}\right)^{12 \cdot 5}$$

↑ interest rate per period, in this case 1 month

total number of periods, in this case, $12 \cdot 5 = 60$ months

$$A = 1000\,(1.00583)^{60}$$

$$A = 1000\,(1.41734)$$

$$\boxed{A = 1417.34}$$

Suppose no compounding $\Rightarrow$ $A = 1000(1+.07)^5$

$$A = 1000(1.40255)$$

$$\boxed{A = 1402.55}$$ Less

SEMIANNUAL ⇒ twice per year OR every 6 months.

Ex 2 Pg562

$24 paid for ISLAND OF MANHATTAN in 1626.

Assume invested at 6% compounded SEMIANNUALLY

How much is $24 worth in 2026

2026
−1626
400 years

$P = 24 \quad N = 2 \quad r = .06$

$$A = P\left(1 + \frac{r}{N}\right)^{Nt}$$

$$A = 24\left(1 + \frac{.06}{2}\right)^{2 \cdot 400}$$

$$A = 24\left(1 + .03\right)^{800}$$

$$A = 24\left(1.03\right)^{800}$$

$$A = 24\left(1.86\,E10\right) \text{ or } 24\left(1.86 \times 10^{10}\right)$$

$$A = 44.64 \times 10^{10}$$

$$A = 4.464 \times 10^{11} = 446{,}400{,}000{,}000$$

$446 Billion

Exponential Decay

Just like growth but use a $\ominus$ r

$$y = C(1-r)^t$$

Ex 3 Pg 562 (A) 1950, Coal use $\Rightarrow$ 114.6 million tons (1.146×10^8)

$r = 6.6\%$ ↓ per year.

Model: $y = 1.146 \times 10^8 (1-.066)^t$

or $y = 1.146 \times 10^8 (.934)^t$

(b) Estimate coal use in 2015 $\Rightarrow t = 65$ years

$y = 1.146 \times 10^8 (.934)^{65}$

$y = 1.146 \times 10^8 (.011819)$

$y = 1.146 \times 10^8 (1.1813 \times 10^{-2})$

$y = 1.3543 \times 10^6$

$y = 1.3543$ million tons

Depreciation ⇒ the rate at which something loses value due to age.

Ex 4 P563

Farmer buys tractor for $50,000
Depreciates 10% per year
Find value in 7 years

$$y = c(1-r)^t$$

$$y = 50000(1-.1)^7$$

$$y = 50000(.9)^7$$

$$y = 50000(.4783)$$

$$y = \$23915$$

Homework: Pg 564 # 11, 12, 14, 15, 16, 17, 19.